**AB/BC Calculus Exam – Review Sheet – Solutions**

## A. Precalculus Type problems

| | When you see the words … | This is what you think of doing |
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| A1 | Find the zeros of $f(x)$. | Set function equal to 0. Factor or use quadratic equation if quadratic. Graph to find zeros on calculator. |
| A2 | Find the intersection of $f(x)$ and $g(x)$. | Set the two functions equal to each other. Find intersection on calculator. |
| A3 | Show that $f(x)$ is even. | Show that $f(-x)=f(x)$. This shows that the graph of $f$ is symmetric to the $y$-axis. |
| A4 | Show that $f(x)$ is odd. | Show that $f(-x)=-f(x)$. This shows that the graph of $f$ is symmetric to the origin. |
| A5 | Find domain of $f(x)$. | Assume domain is $(-\infty,\infty)$. Restrict domains: denominators $\neq$ 0, square roots of only non-negative numbers, logarithm or natural log of only positive numbers. |
| A6 | Find vertical asymptotes of $f(x)$. | Express $f(x)$ as a fraction, express numerator and denominator in factored form, and do any cancellations. Set denominator equal to 0. |
| A7 | If continuous function $f(x)$ has $f(a)<k$ and $f(b)>k$, explain why there must be a value $c$ such that $a<c<b$ and $f(c)=k$. | This is the Intermediate Value Theorem. |

## B. Limit Problems

| | When you see the words … | This is what you think of doing |
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| B1 | Find $\lim\limits_{x\to a} f(x)$. | Step 1: Find $f(a)$. If you get a zero in the denominator,<br>Step 2: Factor numerator and denominator of $f(x)$. Do any cancellations and go back to Step 1. If you still get a zero in the denominator, the answer is either ∞, -∞, or does not exist. Check the signs of $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$ for equality. |
| B2 | Find $\lim\limits_{x\to a} f(x)$ where $f(x)$ is a piecewise function. | Determine if $\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$ by plugging in $a$ to $f(x), x<a$ and $f(x), x>a$ for equality. If they are not equal, the limit doesn't exist. |
| B3 | Show that $f(x)$ is continuous. | Show that 1) $\lim\limits_{x\to a} f(x)$ exists<br>2) $f(a)$ exists<br>3) $\lim\limits_{x\to a} f(x)=f(a)$ |
| B4 | Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$. | Express $f(x)$ as a fraction. Determine location of the highest power:<br>Denominator: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to-\infty} f(x)=0$<br>Both Num and Denom: ratio of the highest power coefficients<br>Numerator: $\lim\limits_{x\to\infty} f(x)=\pm\infty$ (plug in large number) |
| B5 | Find horizontal asymptotes of $f(x)$. | $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$ |

| | When you see the words … | This is what you think of doing |
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| B6 BC | Find $\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)}$ if $\lim\limits_{x\to 0} f(x)=0$ and $\lim\limits_{x\to 0} g(x)=0$ | Use L'Hopital's Rule: $\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to 0}\dfrac{f'(x)}{g'(x)}$ |
| B7 BC | Find $\lim\limits_{x\to 0} f(x)\cdot g(x)=0(\pm\infty)$ | Express $g(x)=\dfrac{1}{\frac{1}{g(x)}}$ and apply L'Hopital's rule. |
| B8 BC | Find $\lim\limits_{x\to 0} f(x)-g(x)=\infty-\infty$ | Express $f(x)-g(x)$ with a common denominator and use L'Hopital's rule. |
| B9 BC | Find $\lim\limits_{x\to 0} f(x)^{g(x)}=1^{\infty}$ or $0^0$ or $\infty^0$ | Take the natural log of the expression and apply L'Hopital's rule, remembering to take the resulting answer and raise *e* to that power. |

**C. Derivatives, differentiability, and tangent lines**

| | When you see the words … | This is what you think of doing |
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| C1 | Find the derivative of a function using the derivative definition. | Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ or $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ |
| C2 | Find the average rate of change of *f* on [*a*, *b*]. | Find $\dfrac{f(b)-f(a)}{b-a}$ |
| C3 | Find the instantaneous rate of change of *f* at $x = a$. | Find $f'(a)$ |
| C4 | Given a chart of *x* and $f(x)$ and selected values of *x* between *a* and *b*, approximate $f'(c)$ where *c* is a value between *a* and *b*. | Straddle *c*, using a value of $k \geq c$ and a value of $h \leq c$. $f'(c)\approx\dfrac{f(k)-f(h)}{k-h}$ |
| C5 | Find the equation of the tangent line to *f* at $(x_1,y_1)$. | Find slope $m = f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$ |
| C6 | Find the equation of the normal line to *f* at $(x_1,y_1)$. | Find slope $m\perp=\dfrac{-1}{f'(x_i)}$. Then use point slope equation: $y-y_1=m(x-x_1)$ |
| C7 | Find *x*-values of horizontal tangents to *f*. | Write $f'(x)$ as a fraction. Set numerator of $f'(x)=0$. |
| C8 | Find *x*-values of vertical tangents to *f*. | Write $f'(x)$ as a fraction. Set denominator of $f'(x)=0$. |
| C9 | Approximate the value of $f(x_1+a)$ if you know the function goes through point $(x_1,y_1)$. | Find slope $m = f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$. Evaluate this line for *y* at $x = x_1 + a$. Note: The closer *a* is to 0, the better the approximation will be. Also note that using concavity, it can be determine if this value is an over or under-approximation for $f(x_1+a)$. |
| C10 | Find the derivative of $f(g(x))$. | This is the chain rule. You are finding $f'(g(x))\cdot g'(x)$. |
| C11 | The line $y = mx + b$ is tangent to the graph of $f(x)$ at $(x_1,y_1)$. | Two relationships are true:<br>1) The function *f* and the line share the same slope at $x_1$: $m = f'(x_1)$<br>2) The function *f* and the line share the same *y*-value at $x_1$. |

| | When you see the words … | This is what you think of doing |
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| C12 | Find the derivative of the inverse to $f(x)$ at $x = a$. | Follow this procedure:<br>1) Interchange $x$ and $y$ in $f(x)$.<br>2) Plug the $x$-value into this equation and solve for $y$ (you may need a calculator to solve graphically)<br>3) Using the equation in 1) find $\frac{dy}{dx}$ implicitly.<br>4) Plug the $y$-value you found in 2) to $\frac{dy}{dx}$ |
| C13 | Given a piecewise function, show it is differentiable at $x = a$ where the function rule splits. | First, be sure that $f(x)$ is continuous at $x = a$. Then take the derivative of each piece and show that $\lim\limits_{x\to a^-} f'(x) = \lim\limits_{x\to a^+} f'(x)$. |

**D. Applications of Derivatives**

| | When you see the words … | This is what you think of doing |
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| D1 | Find critical values of $f(x)$. | Find and express $f'(x)$ as a fraction. Set both numerator and denominator equal to zero and solve. |
| D2 | Find the interval(s) where $f(x)$ is increasing/decreasing. | Find critical values of $f'(x)$. Make a sign chart to find sign of $f'(x)$ in the intervals bounded by critical values. Positive means increasing, negative means decreasing. |
| D3 | Find points of relative extrema of $f(x)$. | Make a sign chart of $f'(x)$. At $x = c$ where the derivative switches from negative to positive, there is a relative minimum. When the derivative switches from positive to negative, there is a relative maximum. To actually find the point, evaluate $f(c)$. OR if $f'(c) = 0$, then if $f''(c) > 0$, there is a relative minimum at $x = c$. If $f''(c) < 0$, there is a relative maximum at $x = c$. ($2^{nd}$ Derivative test). |
| D4 | Find inflection points of $f(x)$. | Find and express $f''(x)$ as a fraction. Set both numerator and denominator equal to zero and solve. Make a sign chart of $f''(x)$. Inflection points occur when $f''(x)$ witches from positive to negative or negative to positive. |
| D5 | Find the absolute maximum or minimum of $f(x)$ on [$a$, $b$]. | Use relative extrema techniques to find relative max/mins. Evaluate $f$ at these values. Then examine $f(a)$ and $f(b)$. The largest of these is the absolute maximum and the smallest of these is the absolute minimum |
| D6 | Find range of $f(x)$ on $(-\infty,\infty)$. | Use relative extrema techniques to find relative max/mins. Evaluate $f$ at these values. Then examine $f(a)$ and $f(b)$. Then examine $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$. |
| D7 | Find range of $f(x)$ on [$a$, $b$] | Use relative extrema techniques to find relative max/mins. Evaluate $f$ at these values. Then examine $f(a)$ and $f(b)$. Then examine $f(a)$ and $f(b)$. |
| D8 | Show that Rolle's Theorem holds for $f(x)$ on [$a$, $b$]. | Show that $f$ is continuous and differentiable on [$a$, $b$]. If $f(a) = f(b)$, then find some $c$ on [$a$, $b$] such that $f'(c) = 0$. |

| | | |
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| D9 | Show that the Mean Value Theorem holds for $f(x)$ on [*a*, *b*]. | Show that *f* is continuous and differentiable on [*a*, *b*]. If $f(a) = f(b)$, then find some *c* on [*a*, *b*] such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ |
| D10 | Given a graph of $f'(x)$, determine intervals where $f(x)$ is increasing/decreasing. | Make a sign chart of $f'(x)$ and determine the intervals where $f'(x)$ is positive and negative. |
| D11 | Determine whether the linear approximation for $f(x_1 + a)$ over-estimates or under-estimates $f(x_1 + a)$. | Find slope $m = f'(x_i)$. Then use point slope equation: $y - y_1 = m(x - x_1)$. Evaluate this line for *y* at $x = x_1 + a$. If $f''(x_1) > 0$, *f* is concave up at $x_1$ and the linear approximation is an underestimation for $f(x_1 + a)$. $f''(x_1) < 0$, *f* is concave down at $x_1$ and the linear approximation is an overestimation for $f(x_1 + a)$. |
| D12 | Find intervals where the slope of $f(x)$ is increasing. | Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$ looking for positive intervals. |
| D13 | Find the minimum slope of $f(x)$ on [*a*, *b*]. | Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$. Values of *x* where $f''(x)$ switches from negative to positive are potential locations for the minimum slope. Evaluate $f'(x)$ at those values and also $f'(a)$ and $f'(b)$ and choose the least of these values. |

**E. Integral Calculus**

| | When you see the words … | This is what you think of doing |
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| E1 | Approximate $\int_a^b f(x)\,dx$ using left Riemann sums with *n* rectangles. | $A = \left(\dfrac{b-a}{n}\right)\left[f(x_0) + f(x_1) + f(x_2) + ... + f(x_{n-1})\right]$ |
| E2 | Approximate $\int_a^b f(x)\,dx$ using right Riemann sums with *n* rectangles. | $A = \left(\dfrac{b-a}{n}\right)\left[f(x_1) + f(x_2) + f(x_3) + ... + f(x_n)\right]$ |
| E3 | Approximate $\int_a^b f(x)\,dx$ using midpoint Riemann sums. | Typically done with a table of points. Be sure to use only values that are given. If you are given 7 points, you can only calculate 3 midpoint rectangles. |
| E4 | Approximate $\int_a^b f(x)\,dx$ using trapezoidal summation. | $A = \left(\dfrac{b-a}{2n}\right)\left[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)\right]$ This formula only works when the base of each trapezoid is the same. If not, calculate the areas of individual trapezoids. |
| E5 | Find $\int_b^a f(x)\,dx$ where $a < b$. | $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ |

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| E6 | Meaning of $\int_a^x f(t)\,dt$. | The accumulation function – accumulated area under function $f$ starting at some constant $a$ and ending at some variable $x$. |
| E7 | Given $\int_a^b f(x)\,dx$, find $\int_a^b \left[f(x)+k\right]dx$. | $\int_a^b \left[f(x)+k\right]dx = \int_a^b f(x)\,dx + \int_a^b k\,dx$ |
| E8 | Given the value of $F(a)$ where the antiderivative of $f$ is $F$, find $F(b)$. | Use the fact that $\int_a^b f(x)\,dx = F(b)-F(a)$ so $F(b) = F(a) + \int_a^b f(x)\,dx$. Use the calculator to find the definite integral. |
| E9 | Find $\frac{d}{dx}\int_a^x f(t)\,dt$. | $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. The 2nd Fundamental Theorem. |
| E10 | Find $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt$. | $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f\left(g(x)\right)\cdot g'(x)$. The 2nd Fundamental Theorem. |
| E11 BC | Find $\int_0^\infty f(x)\,dx$. | $\int_0^\infty f(x)\,dx = \lim\limits_{h\to\infty}\int_0^h f(x)\,dx = \lim\limits_{h\to\infty} F(h) - F(0)$. |
| E12 BC | Find $\int f(x)\cdot g(x)\,dx$ | If $u$-substitution doesn't work, try integration by parts: $\int u\cdot dv = uv - \int v\cdot du$ |

## F. Applications of Integral Calculus

| | When you see the words … | This is what you think of doing |
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| F1 | Find the area under the curve $f(x)$ on the interval $[a, b]$. | $\int_a^b f(x)\,dx$ |
| F2 | Find the area between $f(x)$ and $g(x)$. | Find the intersections, $a$ and $b$ of $f(x)$ and $g(x)$. If $f(x) \geq g(x)$ on $[a,b]$, then area $A = \int_a^b \left[f(x)-g(x)\right]dx$. |
| F3 | Find the line $x = c$ that divides the area under $f(x)$ on $[a, b]$ into two equal areas. | $\int_a^c f(x)\,dx = \int_c^b f(x)\,dx$ or $\int_a^b f(x)\,dx = 2\int_a^c f(x)\,dx$ |
| F4 | Find the volume when the area under $f(x)$ is rotated about the $x$-axis on the interval $[a, b]$. | Disks: Radius = $f(x)$: $V = \pi\int_a^b \left[f(x)\right]^2 dx$ |
| F5 | Find the volume when the area between $f(x)$ and $g(x)$ is rotated about the $x$-axis. | Washers: Outside radius = $f(x)$. Inside radius = $g(x)$. Establish the interval where $f(x) \geq g(x)$ and the values of $a$ and $b$, where $f(x) = g(x)$. $V = \pi\int_a^b \left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$ |

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| F6 | Given a base bounded by $f(x)$ and $g(x)$ on [$a$, $b$] the cross sections of the solid perpendicular to the $x$-axis are squares. Find the volume. | Base = $f(x)-g(x)$. Area = base$^2$ = $\left[f(x)-g(x)\right]^2$. Volume = $\int_a^b \left[f(x)-g(x)\right]^2 dx$ |
| F7 | Solve the differential equation $\frac{dy}{dx}=f(x)g(y)$. | Separate the variables: $x$ on one side, $y$ on the other with the $dx$ and $dy$ in the numerators. Then integrate both sides, remembering the +$C$, usually on the $x$-side. |
| F8 | Find the average value of $f(x)$ on [$a$, $b$]. | $F_{avg}=\dfrac{\int_a^b f(x)\,dx}{b-a}$ |
| F9 | Find the average rate of change of $F'(x)$ on $[t_1,t_2]$. | $\dfrac{\frac{d}{dt}\int_{t_1}^{t_2} F'(x)\,dx}{t_2-t_1}=\dfrac{F'(t_2)-F'(t_1)}{t_2-t_1}$ |
| F10 | $y$ is increasing proportionally to $y$. | $\frac{dy}{dt}=ky$ which translates to $y=Ce^{kt}$ |
| F11 | Given $\frac{dy}{dx}$, draw a slope field. | Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the calculated slopes at the point. |
| F12 BC | Find $\int \frac{dx}{ax^2+bx+c}$ | Factor $ax^2+bx+c$ into non-repeating factors to get $\int \frac{dx}{(mx+n)(px+q)}$ and use Heaviside method to create partial fractions and integrate each fraction. |
| F13 BC | Use Euler's method to approximate $f(1.2)$ given a formula for $\frac{dy}{dx}$, $(x_0,y_0)$ and $\Delta x=0.1$ | $dy=\frac{dy}{dx}(\Delta x)$, $y_{\text{new}}=y_{\text{old}}+dy$ |
| F14 BC | Is the Euler's approximation an over- or under-approximation? | Look at sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the interval. This gives increasing/decreasing and concavity information. Draw a picture to ascertain the answer. |
| F15 BC | A population $P$ is increasing logistically. | $\frac{dP}{dt}=kP(C-P)$. |
| F16 BC | Find the carrying capacity of a population growing logistically. | $\frac{dP}{dt}=kP(C-P)=0\Rightarrow C=P$. |
| F17 BC | Find the value of $P$ when a population growing logistically is growing the fastest. | $\frac{dP}{dt}=kP(C-P)\Rightarrow$ Set $\frac{d^2P}{dt^2}=0$ |
| F18 BC | Given continuous $f(x)$, find the arc length on [$a$, $b$] | $L=\int_a^b\sqrt{1+\left[f'(x)\right]^2}\,dx$ |

**G. Particle Motion and Rates of Change**

| | When you see the words … | This is what you think of doing |
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| G1 | Given the position function $s(t)$ of a particle moving along a straight line, find the velocity and acceleration. | $v(t) = s'(t) \qquad a(t) = v'(t) = s''(t)$ |
| G2 | Given the velocity function $v(t)$ and $s(0)$, find $s(t)$. | $s(t) = \int v(t)\,dt + C$. Plug in $s(0)$ to find $C$. |
| G3 | Given the acceleration function $a(t)$ of a particle at rest and $s(0)$, find $s(t)$. | $v(t) = \int a(t)\,dt + C_1$. Plug in $v(0) = 0$ to find $C_1$.<br>$s(t) = \int v(t)\,dt + C_2$. Plug in $s(0)$ to find $C_2$. |
| G4 | Given the velocity function $v(t)$, determine if a particle is speeding up or slowing down at $t = k$. | Find $v(k)$ and $a(k)$. If both have the same sign, the particle is speeding up. If they have different signs, the particle is slowing down. |
| G5 | Given the position function $s(t)$, find the average velocity on $[t_1, t_2]$. | Avg. vel. $= \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$ |
| G6 | Given the position function $s(t)$, find the instantaneous velocity at $t = k$. | Inst. vel. $= s'(k)$. |
| G7 | Given the velocity function $v(t)$ on $[t_1, t_2]$, find the minimum acceleration of a particle. | Find $a(t)$ and set $a'(t) = 0$. Set up a sign chart and find critical values. Evaluate the acceleration at critical values and also $t_1$ and $t_2$ to find the minimum. |
| G8 | Given the velocity function $v(t)$, find the average velocity on $[t_1, t_2]$. | Avg. vel. $= \dfrac{\int_{t_1}^{t_2} v(t)\,dt}{t_2 - t_1}$ |
| G9 | Given the velocity function $v(t)$, determine the difference of position of a particle on $[t_1, t_2]$. | Displacement $= \int_{t_1}^{t_2} v(t)\,dt$ |
| G10 | Given the velocity function $v(t)$, determine the distance a particle travels on $[t_1, t_2]$. | Distance $= \int_{t_1}^{t_2} \lvert v(t) \rvert\,dt$ |
| G11 | Calculate $\int_{t_1}^{t_2} \lvert v(t) \rvert\,dt$ without a calculator. | Set $v(t) = 0$ and make a sign charge of $v(t) = 0$ on $[t_1, t_2]$. On intervals [*a*, *b*] where $v(t) > 0$, $\int_a^b \lvert v(t) \rvert\,dt = \int_a^b v(t)\,dt$<br>On intervals [*a*, *b*] where $v(t) < 0$, $\int_a^b \lvert v(t) \rvert\,dt = \int_b^a v(t)\,dt$ |
| G12 | Given the velocity function $v(t)$ and $s(0)$, find the greatest distance of the particle from the starting position on $[0, t_1]$. | Generate a sign chart of $v(t)$ to find turning points.<br>$s(t) = \int v(t)\,dt + C$. Plug in $s(0)$ to find $C$.<br>Evaluate $s(t)$ at all turning points and find which one gives the maximum distance from $s(0)$. |

| | When you see the words … | This is what you think of doing |
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| G13 | The volume of a solid is changing at the rate of … | $\frac{dV}{dt} = \ldots$ |
| G14 | The meaning of $\int_a^b R'(t)\,dt$. | This gives the accumulated change of $R(t)$ on [$a$, $b$]. $\int_a^b R'(t)\,dt = R(b) - R(a)$ or $R(b) = R(a) + \int_a^b R'(t)\,dt$ |
| G15 | Given a water tank with $g$ gallons initially, filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$ a) The amount of water in the tank at $t = m$ minutes. b) the rate the water amount is changing at $t = m$ minutes and c) the time $t$ when the water in the tank is at a minimum or maximum. | a) $g + \int_0^m [F(t) - E(t)]\,dt$<br>b) $\frac{d}{dt}\int_0^m [F(t) - E(t)]\,dt = F(m) - E(m)$<br>c) set $F(m) - E(m) = 0$, solve for $m$, and evaluate $g + \int_0^m [F(t) - E(t)]\,dt$ at values of $m$ and also the endpoints. |

**H. Parametric and Polar Equations - BC**

| | When you see the words … | This is what you think of doing |
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| H1 | Given $x = f(t), y = g(t)$, find $\frac{dy}{dx}$. | $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ |
| H2 | Given $x = f(t), y = g(t)$, find $\frac{d^2y}{dx^2}$. | $x = f(t), y = g(t)$, find $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ |
| H3 | Given $x = f(t), y = g(t)$, find arc length on $[t_1, t_2]$. | $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ |
| H4 | Express a polar equation in the form of $r = f(\theta)$ in parametric form. | $x = r\cos\theta = f(\theta)\cos\theta \quad y = r\sin\theta = f(\theta)\sin\theta$ |
| H5 | Find the slope of the tangent line to $r = f(\theta)$. | $x = r\cos\theta \quad y = r\sin\theta \Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ |
| H6 | Find horizontal tangents to a polar curve $r = f(\theta)$. | $x = r\cos\theta \quad y = r\sin\theta$<br>Find where $r\sin\theta = 0$ when $r\cos\theta \neq 0$ |
| H7 | Find vertical tangents to a polar curve $r = f(\theta)$. | $x = r\cos\theta \quad y = r\sin\theta$<br>Find where $r\cos\theta = 0$ when $r\sin\theta \neq 0$ |
| H8 | Find the area bounded by the polar curve $r = f(\theta)$ on $[\theta_1, \theta_2]$. | $A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} [f(\theta)]^2\,d\theta$ |
| H9 | Find the arc length of the polar curve $r = f(\theta)$ on $[\theta_1, \theta_2]$. | $s = \int_{\theta_1}^{\theta_2} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\,d\theta = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ |

## I. Vectors and Vector-valued functions - BC

| | When you see the words … | This is what you think of doing |
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| I1 | Find the magnitude of vector $v\langle v_1, v_2\rangle$. | $\lVert v\rVert = \sqrt{v_1^{\,2} + v_2^{\,2}}$ |
| I2 | Find the dot product: $\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle$ | $\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle = u_1 v_1 + u_2 v_2$ |
| I3 | The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t)\rangle$. Find a) the velocity vector and b) the acceleration vector. | a) $v(t) = \langle x'(t), y'(t)\rangle$<br>b) $a(t) = \langle x''(t), y''(t)\rangle$ |
| I4 | The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t)\rangle$. Find the speed of the particle at time *t*. | Speed $= \lVert v(t)\rVert = \sqrt{[x'(t)]^2 + [y'(t)]^2}$ - a scalar |
| I5 | Given the velocity vector $v(t) = \langle x(t), y(t)\rangle$ and position at time $t = 0$, find the position vector. | $s(t) = \int x(t)\,dt + \int y(t)\,dt + C$<br>Use $s(0)$ to find *C*, remembering that it is a vector. |
| I6 | Given the velocity vector $v(t) = \langle x(t), y(t)\rangle$, when does the particle stop? | $v(t) = 0 \Rightarrow x(t) = 0$ AND $y(t) = 0$ |
| I7 | The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t)\rangle$. Find the distance the particle travels from $t_1$ to $t_2$. | Distance $= \int_{t_1}^{t_2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$ |

## J. Taylor Polynomial Approximations - BC

| | When you see the words … | This is what you think of doing |
|---|---|---|
| J1 | Find the *n*th degree Maclaurin polynomial to $f(x)$. | $P_n(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \dfrac{f'''(0)}{3!}x^3 + \ldots + \dfrac{f^{(n)}(0)}{n!}x^n$ |
| J2 | Find the *n*th degree Taylor polynomial to $f(x)$ centered at $x = c$. | $P_n(x) = f(c) + f'(c)(x-c) + \dfrac{f''(c)(x-c)^2}{2!} + \dfrac{f'''(c)(x-c)^3}{3!} + \ldots + \dfrac{f^{(n)}(c)(x-c)^n}{n!}$ |
| J3 | Use the first-degree Taylor polynomial to $f(x)$ centered at $x = c$ to approximate $f(k)$ and determine whether the approximation is greater than or less than $f(k)$. | Write the first-degree TP and find $f(k)$. Use the signs of $f'(c)$ and $f''(c)$ to determine increasing/decreasing and concavity and draw your line (1st degree TP) to determine whether the line is under the curve (under-approximation) or over the curve (over-approximation). |

| | When you see the words … | This is what you think of doing |
|---|---|---|
| J4 | Given an $n$th degree Taylor polynomial for $f$ about $x = c$, find $f(c), f'(c), f''(c), \ldots, f^{(n)}(c)$ | $f(c)$ will be the constant term in your Taylor polynomial (TP)<br>$f'(c)$ will be the coefficient of the $x$ term in the TP.<br>$\frac{f''(c)}{2!}$ will be the coefficient of the $x^2$ term in the TP.<br>$\frac{f^{(n)}(c)}{n!}$ will be the coefficient of the $x^n$ term in the TP. |
| J5 | Given a Taylor polynomial centered at $c$, determine if there is enough information to determine if there is a relative maximum or minimum at $x = c$. | If there is no first-degree $x$-term in the TP, then the value of $c$ about which the function is centered is a critical value. Thus the coefficient of the $x^2$ term is the second derivative divided by 2! Using the second derivative test, we can tell whether there is a relative maximum, minimum, or neither at $x = c$. |
| J6 | Given an $n$th degree Taylor polynomial for $f$ about $x = c$, find the Lagrange error bound (remainder). | $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}\|x-c\|^{n+1}$. The value of $z$ is some number between $x$ and $c$. $f^{(n+1)}(z)$ represents the $(n+1)^{\text{st}}$ derivative of $z$. This usually is given to you. |
| J7 | Given an $n$th degree Maclaurin polynomial $P$ for $f$, find the $\|f(k) - P(k)\|$. | This is looking for the Lagrange error – the difference between the value of the function at $x = k$ and the value of the TP at $x = k$. |

**K. Infinite Series - BC**

| | When you see the words … | This is what you think of doing |
|---|---|---|
| K1 | Given $a_n$, determine whether the sequence $a_n$ converges. | $a_n$ converges if $\lim\limits_{n\to\infty} a_n$ exists. |
| K2 | Given $a_n$, determine whether the series $a_n$ could converge. | If $\lim\limits_{n\to\infty} a_n = 0$, the series could converge. If $\lim\limits_{n\to\infty} a_n \neq 0$, the series cannot converge. ($n$th term test). |
| K3 | Determine whether a series converges. | Examine the $n$th term of the series. Assuming it passes the $n$th term test, the most widely used series forms and their rule of convergence are:<br>Geometric: $\sum_{n=0}^{\infty} ar^n$ - converges if $\|r\| < 1$<br>$p$-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ - converges if $p > 1$<br>Alternating: $\sum_{n=1}^{\infty} (-1)^n a_n$ - converges if $0 < a_{n+1} < a_n$<br>Ratio: $\sum_{n=0}^{\infty} a_n$ - converges if $\lim\limits_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| < 1$ |
| K4 | Find the sum of a geometric series. | $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ |
| K5 | Find the interval of convergence of a series. | If not given, you will have to generate the $n$th term formula. Use a test (usually the ratio test) to find the interval of convergence and then check out the endpoints. |

| | When you see the words … | This is what you think of doing |
|---|---|---|
| K6 | $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$ | The harmonic series – divergent. |
| K7 | $f(x)=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$ | $f(x)=e^x$ |
| K8 | $f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...$ | $f(x)=\sin x$ |
| K9 | $f(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...$ | $f(x)=\cos x$ |
| K10 | $f(x)=1+x+x^2+x^3+...+x^n+....$ | $f(x)=\frac{1}{1-x}$ Convergent : $(-1,1)$ |
| K11 | Given a formula for the *n*th derivative of $f(x)$. Write the first four terms and the general term for the power series for $f(x)$ centered at $x=c$. | $f(x)=f(c)+f'(c)(x-c)+\frac{f''(c)(x-c)^2}{2!}+\frac{f'''(c)(x-c)^3}{3!}+...+\frac{f^{(n)}(c)(x-c)^n}{n!}+...$ |
| K12 | Let $S_4$ be the sum of the first 4 terms of an alternating series for $f(x)$. Approximate $\lvert f(x)-S_4\rvert$. | This is the error for the 4th term of an alternating series which is simply the 5th tern. It will be positive since you are looking for an absolute value. |
| K13 | Write a series for expressions like $e^{x^2}$. | Rather than go through generating a Taylor polynomial, use the fact that if $f(x)=e^x$, then $f(x^2)=e^{x^2}$. So $f(x)=e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\frac{x^4}{4!}+...+\frac{x^n}{n!}+...$ and $f(x^2)=e^{x^2}=1+x^2+\frac{x^4}{2}+\frac{x^6}{3!}+\frac{x^8}{4!}+...+\frac{x^{2n}}{n!}+...$ |