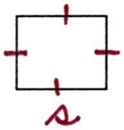
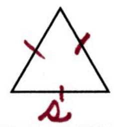
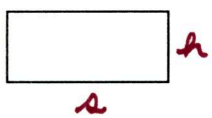
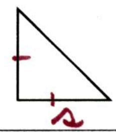
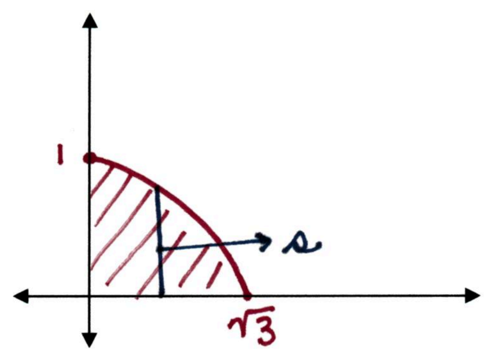
AB Calculus - Notes Name Key
Cross Sectional Volume

Write a formula for the area of:

Square of side s	Equilateral triangle of side s
 $A = s^2$	 $A = s^2 \cdot \frac{\sqrt{3}}{4}$
Rectangle	Isosceles right triangle with leg s (means leg is on the base of solid figure)
 $A = s \cdot h$	 $A = \frac{s^2}{2}$
Isosceles right triangle with hypotenuse s (means hypotenuse is on the base of solid figure)	Semi-circle with radius r (s is the diameter)
 $A = \frac{s \cdot \frac{s}{2}}{2} = \frac{s^2}{4}$	 $A = \frac{\pi \cdot \left(\frac{s}{2}\right)^2}{2} = s^2 \cdot \frac{\pi}{8}$

Example 1: Emanuel the Duck just bought land with a perimeter set by $y = -\frac{1}{3}x^2 + 1$, $x \geq 0$ and $y \geq 0$

He plans to build a Biodome, which uses the area described above as a base. The Biodome will be built up so that cross-sections perpendicular to the x-axis will be squares. He wants to know if he will have enough volume in his biodome to have a party with all his friends. He needs 1 cubic mile of space. What is the volume and will he have enough space to entertain his friends?



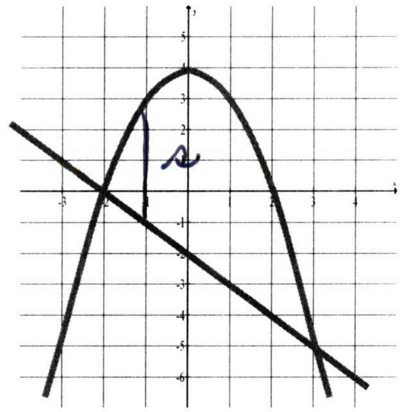
Squares: $A = s^2$

$s = \left(-\frac{1}{3}x^2 + 1\right) - 0$

$$\text{Volume} = \int_0^{\sqrt{3}} \left(-\frac{1}{3}x^2 + 1\right)^2 dx = .9237\ldots \text{ mi}^3$$

He does not have enough space. :(

Example 2: Find the volume of the solid whose base is bounded by the equations $y=-x^2+4$ and $y=-x-2$ and whose cross sections taken perpendicular to the x-axis are:



a. Squares

$$\int_{-2}^{3} s^2\,dx$$

b. Equilateral Triangles

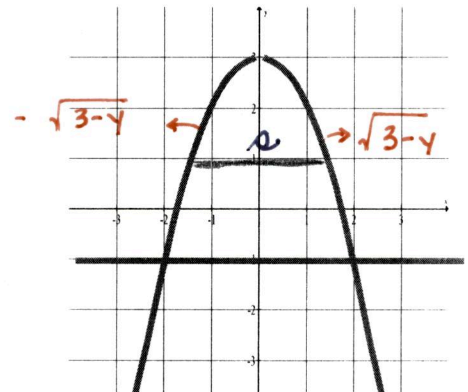
$$\frac{\sqrt{3}}{4}\int_{-2}^{3} s^2\,dx$$

c. Rectangles of height that is twice the base

$$\int_{-2}^{3} s\cdot 2s\,dx$$

$$s=(-x^2+4)-(-x-2)$$

Example 3: Find the volume of the solid whose base is bounded by the equations $y=-x^2+3$ and $y=-1$ and whose cross sections taken perpendicular to the y-axis are:



a. Isosceles Right triangles with the hypotenuse on the base.

$$\frac{1}{4}\int_{-1}^{3} s^2\,dy$$

b. Semicircles

$$\frac{\pi}{8}\int_{-1}^{3} s^2\,dy$$

$$s=\sqrt{3-y}-(-\sqrt{3-y})$$
$$=2\sqrt{3-y}$$

Example 3: (Calculator)

$x=\frac{y}{4}$

Let R be the region in the first quadrant enclosed by the graphs of $y=e^{(x^2)}-1$ and $y=4x$.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

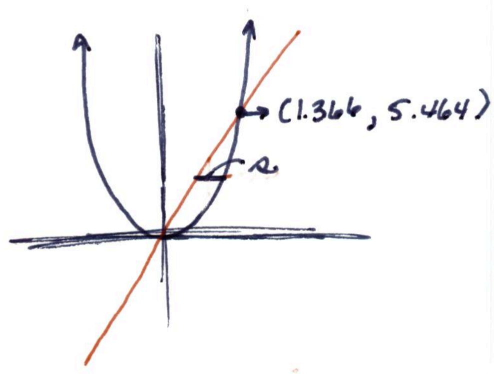
$$\int_{0}^{5.464} s^2\,dy$$

$$=.970$$

$\ln(y+1)=\ln e^{x^2}$

$\ln(y+1)=x^2$

$x=\sqrt{\ln(y+1)}$



$$s=\frac{y}{4}-\sqrt{\ln(y+1)}$$

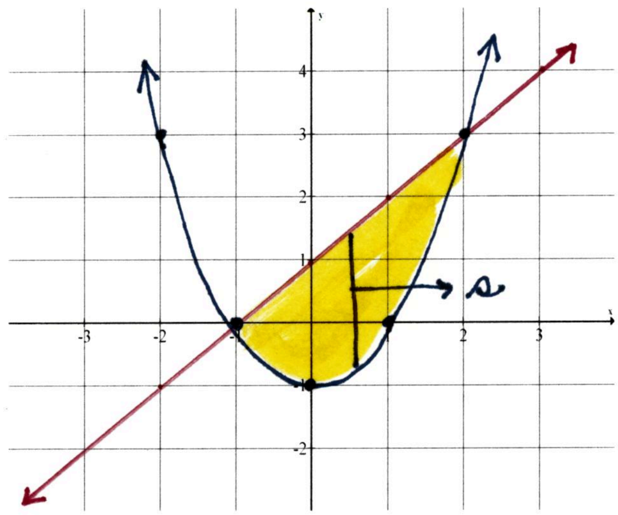
AB Calculus - ~~Notes~~ HW **Name** Key

Cross Sectional Volume – Calculator Active

Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$ with the indicated cross sections taken perpendicular to the x-axis.

1. Squares

$$\int_{-1}^{2} (-x^2 + x + 2)^2 \, dx = 8.1$$



$$s = (x+1) - (x^2 - 1)$$
$$= -x^2 + x + 2$$

2. Isosceles Triangles with a leg on the base.

$$\frac{1}{2} \int_{-1}^{2} s^2 \, dx = 4.05$$

3. Semicircles

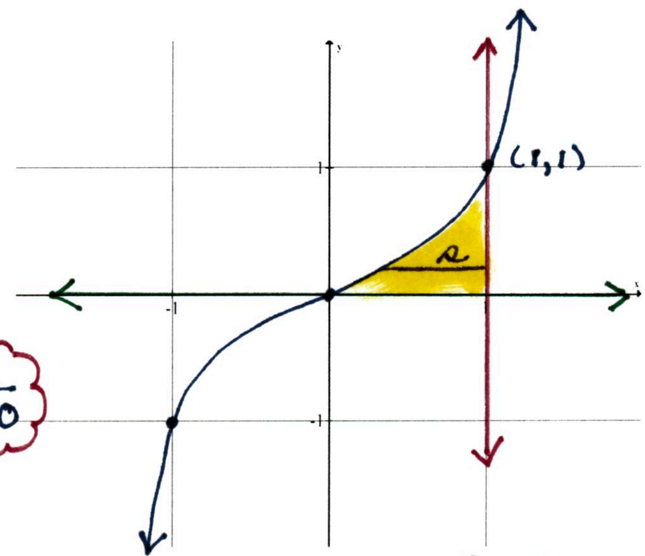
$$\frac{\pi}{8} \int_{-1}^{2} s^2 \, dx = 3.180 \text{ or } \frac{81\pi}{80}$$

4. Equilateral triangles

$$\frac{\sqrt{3}}{4} \int_{-1}^{2} s^2 \, dx = 3.507 \text{ or } \frac{81\sqrt{3}}{40}$$

* By hand

Find the volume of the solid whose base is bounded by the graphs of $y = x^3$ and $y = 0$ and $x = 1$ with the indicated cross sections taken perpendicular to the y-axis.



$s = 1 - \sqrt[3]{y}$

5. Squares

$$\int_0^1 (1-\sqrt[3]{y})^2\,dy =$$

$$\int_0^1 (1-2y^{\frac{1}{3}}+y^{2/3})\,dy =$$

$$y - \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{5}y^{5/3}\Big|_0^1 = 1-\frac{3}{2}+\frac{3}{5} = \frac{1}{10}$$

6. Rectangles of height 1

$$\int_0^1 (1-\sqrt[3]{y})\cdot 1\,dy = \int_0^1 (1-y^{\frac{1}{3}})\,dy$$

$$y - \frac{3}{4}y^{4/3}\Big|_0^1 = 1-\frac{3}{4} = \frac{1}{4}$$

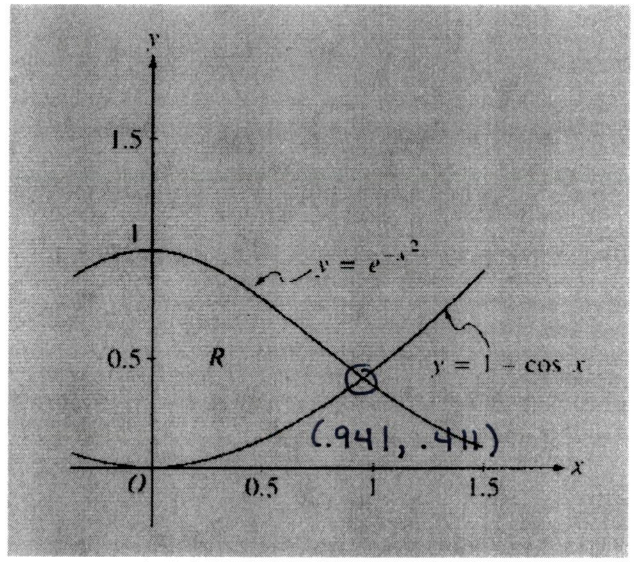
7. Semicircles

$$\frac{\pi}{8}\int_0^1 (1-\sqrt[3]{y})^2\,dy = \frac{\pi}{80}$$

8. Equilateral triangles

$$\frac{\sqrt{3}}{4}\int_0^1 (1-\sqrt[3]{y})^2\,dy = \frac{\sqrt{3}}{40}$$

*2000 AB 1 and BC 1



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

$$\int_0^{.941} \left[e^{-x^2} - (1-\cos x)\right] dx$$

$= .5909$

(b) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$$s = e^{-x^2} - (1-\cos x)$$

$$\int_0^{.941} s^2\, dx = .4610$$