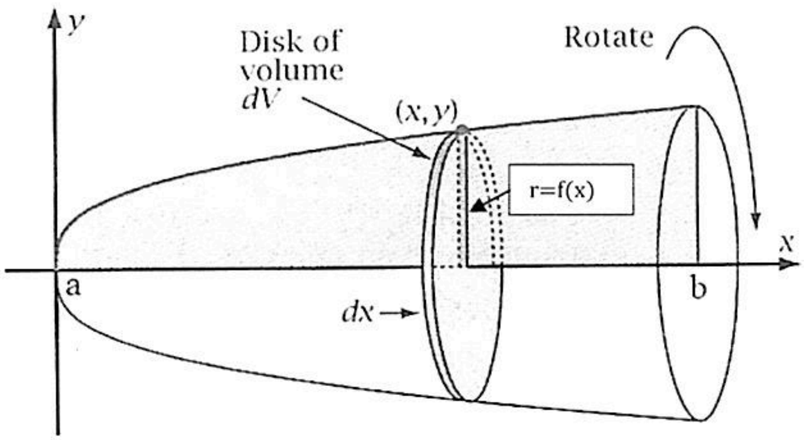
AP Calculus Name Key

Notes: Volume: Disk and Washer Methods
(rotations around an axis)



$$V = \lim_{n \to \infty} \sum_{i=1}^{n} \pi (r_i)^2 \Delta x = \pi \int_a^b [f(x)]^2 dx$$

We are basically finding the sum of the volume of an infinite number of disks. Each disk has a radius of f(x), and a width of dx.

If a region in the plane is revolved about a line, the resulting solid is a solid of revolution, and the line is called the axis of revolution.

The Disk Method

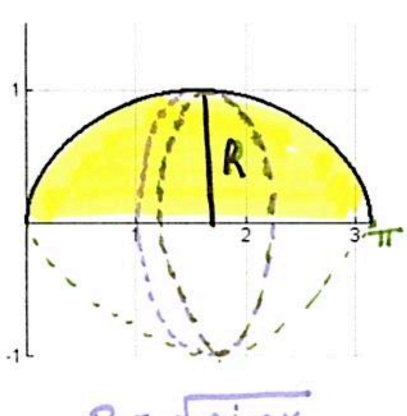
Horizontal Axis of Revolution

$$V = \pi \int_a^b [R(x)]^2 dx$$

Vertical Axis of Revolution

$$V = \pi \int_c^d [R(y)]^2 dy$$

1) Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis ($0 \le x \le \pi$) about the x-axis.



$R = \sqrt{\sin x}$

$V = \pi \int_0^\pi (\sqrt{\sin x})^2 dx = 2\pi$

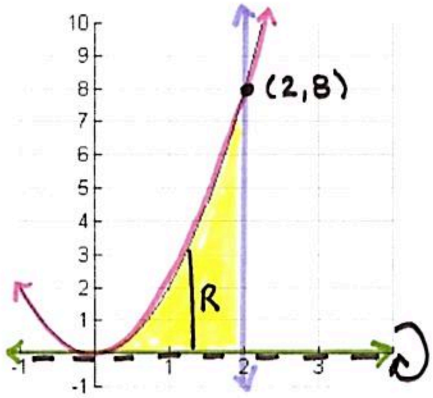
$= \pi \int_0^\pi \sin x \, dx$

$= -\pi \cos x \Big|_0^\pi$

$= [-\pi(\cos \pi)] - [-\pi(\cos 0)]$

$= \pi + \pi$

*2) Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2$, $y = 0$ and $x = 2$ about the x-axis.



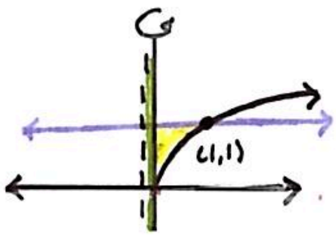
$R = 2x^2$

$$\pi \int_0^2 (2x^2)^2\, dx = \pi \int_0^2 4x^4\, dx$$

$$= \pi \cdot \frac{4x^5}{5} \Big|_0^2$$

$$= \frac{128\pi}{5}$$

3) Find the volume of the solid generated by revolving the region bounded by the graphs $x = y^2$, $x = 0$, and $y = 1$ about the y-axis.

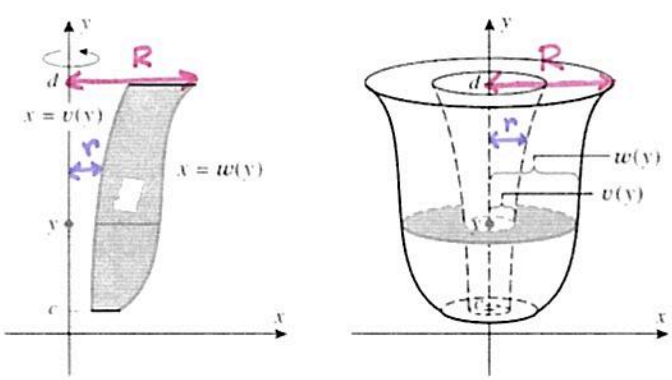


$$\pi \int_0^1 (y^2)^2\, dy = \pi \int_0^1 y^4\, dy$$

$$= \pi \frac{y^5}{5} \Big|_0^1$$

$$= \frac{\pi}{5}$$

Washer Method:



(a) (b)

Washers

$$V = \pi \int_a^b [R(x)]^2\, dx - \pi \int_a^b [r(x)]^2\, dx$$

Or

$$V = \pi \int_a^b \left[[R(x)]^2 - [r(x)]^2 \right] dx$$

You will subtract outer - inner, not top - bottom.

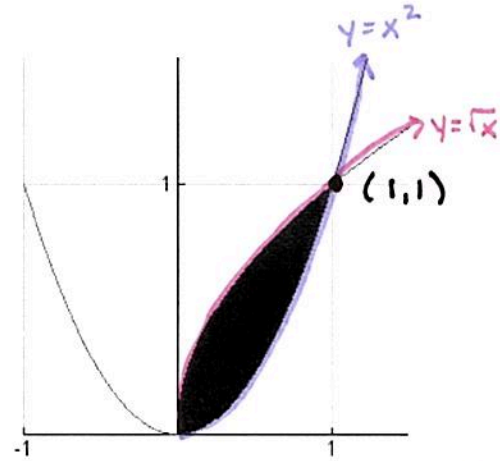
Horizontal Axis of Revolution

$$V = \pi \int_a^b [R(x)]^2\, dx - \pi \int_a^b [r(x)]^2\, dx$$

Vertical Axis of Revolution

$$V = \pi \int_c^d [R(y)]^2\, dy - \pi \int_c^d [r(y)]^2\, dy$$

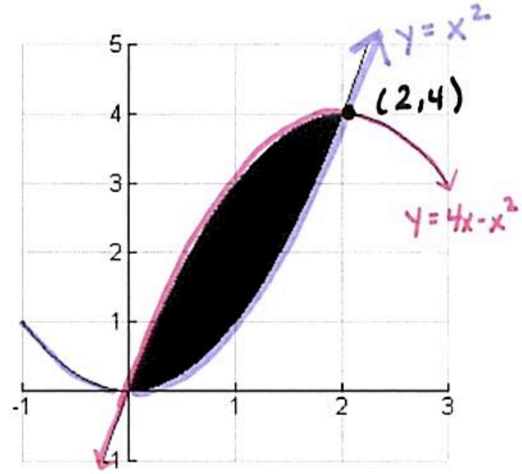
4) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis.



$R = \sqrt{x}$ $\qquad$ $r = x^2$

$$\pi \int_0^1 (\sqrt{x})^2\,dx - \pi \int_0^1 (x^2)^2\,dx$$

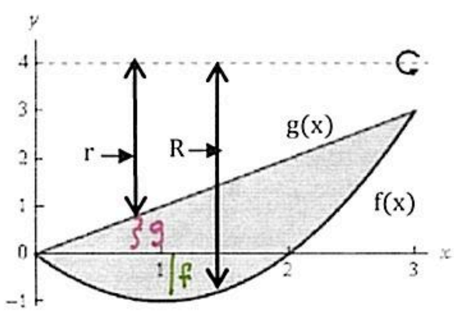
5) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2$ and $y = 4x - x^2$ about the x-axis.



$R = (4x - x^2)$ $\qquad$ $r = x^2$

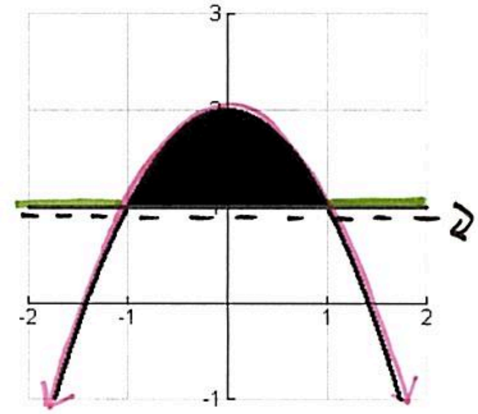
$$\pi \int_0^2 (4x-x^2)^2\,dx - \pi \int_0^2 (x^2)^2\,dx$$

Rotations off the axis:



R = [f(x) – 4] or [4 – f(x)]
r = [g(x) – 4] or [4 – g(x)]

6) Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and g(x) = 1 about the line y = 1.

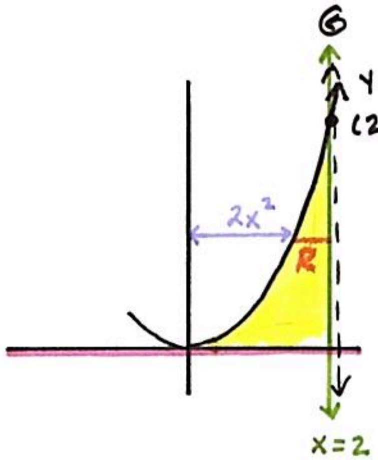


$R = (2 - x^2) - 1$ or $1 - (2 - x^2)$

$$\pi \int_{-1}^{1} [(2-x^2)-1]^2\,dx$$

7) Find the volume of the solid formed by revolving the region bounded by $y = 2x^2$, $y = 0$ and $x = 2$ around the line $x = 2$.

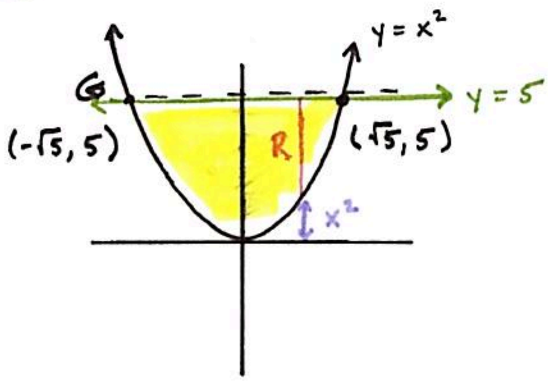
$x = \pm\sqrt{\frac{y}{2}}$



$R = \left(2 - \sqrt{\frac{y}{2}}\right)$ or $\left(\sqrt{\frac{y}{2}} - 2\right)$

$$\pi \int_0^8 \left[\left(2 - \sqrt{\frac{y}{2}}\right)^2\right] dy$$

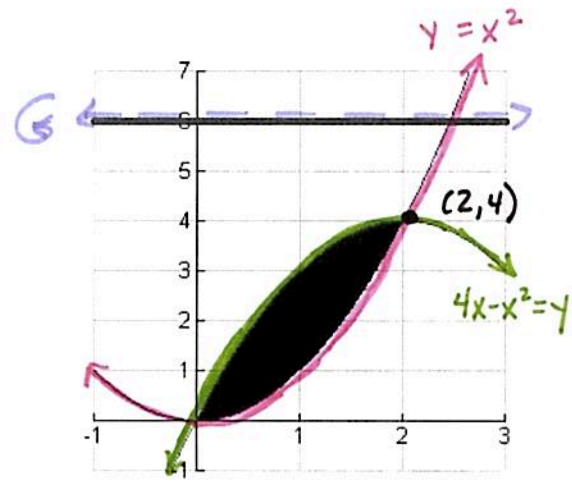
8) Find the volume of the solid formed by revolving the region bounded by $y = x^2$ and $y = 5$ around the line $y = 5$.



$R = (5 - x^2)$ or $(x^2 - 5)$

$$\pi \int_{-\sqrt{5}}^{\sqrt{5}} (5 - x^2)^2 \, dx$$

9) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2$ and $y = 4x - x^2$ about the line $y = 6$.



$R = (6 - x^2)$ or $(x^2 - 6)$

$r = (6 - (4x - x^2))$ or $(4x - x^2 - 6)$

$$\pi \int_0^2 (6 - x^2)^2 \, dx - \pi \int_0^2 [6 - (4x - x^2)] \, dx$$