

Usub Day 1: hwk

$$1. \int x^2 \cos(x^3) dx = \frac{1}{3} \int \cos u du$$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

$$= \frac{1}{3} \sin(x^3) + C$$

$$2. \int \sqrt{\tan x} \cdot \sec^2 x dx = \int u^{1/2} du$$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$dx = \frac{du}{\sec^2 x}$$

$$= \frac{2}{3} \tan^{3/2} x + C$$

$$3. \int x(x^2+1)^3 dx = \frac{1}{2} \int u^3 du$$

$$u = x^2+1$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= \frac{1}{2} \cdot \frac{(x^2+1)^4}{4} + C$$

$$= \frac{(x^2+1)^4}{8} + C$$

$$4. \int \frac{x}{\sqrt{9+x^2}} dx = \frac{1}{2} \int u^{-1/2} du$$

$$u = 9+x^2$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= \frac{1}{2} (2 \cdot (9+x^2)^{1/2}) + C$$

$$= \sqrt{9+x^2} + C$$

$$5. \int \csc(5x) \cot(5x) dx = \frac{1}{5} \int \csc u \cot u du$$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$dx = \frac{du}{5}$$

$$= -\frac{1}{5} \csc(5x) + C$$

$$6. \int (8x^3 - 9x^2 + 5x^{-2}) dx = \frac{8x^4}{4} - \frac{9x^3}{3} + \frac{5x^{-1}}{-1} + C$$

$$= 2x^4 - 3x^3 - \frac{5}{x} + C$$

$$7. \int (2x^2 - 5x - 12) dx = \frac{2x^3}{3} - \frac{5x^2}{2} - 12x + C$$

$$3. \int (x^{-2} + 2x^{-3} - 3x^{-4}) dx = -x^{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C$$

$$= -\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$$

$$1. \int \sin^4(3x) \cos(3x) dx = \frac{1}{3} \int u^4 du$$

$$u = \sin 3x$$

$$\frac{du}{dx} = 3 \cos 3x$$

$$dx = \frac{du}{3 \cos 3x}$$

$$= \frac{1}{3} \cdot \frac{(\sin(3x))^5}{5} + C$$

$$= \frac{\sin^5(3x)}{15} + C$$

$$10. \int_1^2 x(x^2+1)^3 dx = \frac{1}{2} \int_2^5 u^3 du$$

$$u = x^2 + 1$$

$$= \frac{1}{2} \cdot \frac{u^4}{4} \Big|_2^5$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= \frac{5^4}{8} - \frac{2^4}{8}$$

$$\text{if } x=1, u=2$$

$$\text{if } x=2, u=5$$

$$= \frac{625 - 16}{8} = \frac{609}{8}$$

$$11. \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx = \int_0^1 u^2 du$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

$$= \frac{u^3}{3} \Big|_0^1$$

$$= \frac{1}{3} - 0$$

$$\text{if } x=0, u=0$$

$$\text{if } x=\frac{\pi}{2}, u=1$$

$$= \frac{1}{3}$$

$$12. \int_1^4 \frac{e^{\frac{4}{x}}}{x^2} dx = \int_4^1 \frac{e^u}{x^2} \cdot \frac{-x^2 du}{4}$$

$$u = \frac{4}{x} = 4x^{-1}$$

$$= -\frac{1}{4} \int_4^1 e^u du$$

$$\frac{du}{dx} = -\frac{4}{x^2}$$

$$dx = \frac{-x^2 du}{4}$$

$$= -\frac{1}{4} e^u \Big|_4^1$$

$$\text{if } x=1, u=4$$

$$\text{if } x=4, u=1$$

$$= -\frac{1}{4}(e - e^4)$$

$$13. \int_0^1 e^{-2x} dx = \frac{-1}{2} \int_0^{-2} e^u du$$

$$u = -2x$$

$$\frac{du}{dx} = -2$$

$$dx = \frac{du}{-2}$$

$$= -\frac{1}{2} (e^u \Big|_0^{-2})$$

$$= -\frac{1}{2} (e^{-2} - e^0)$$

$$\text{if } x=0, u=0$$

$$\text{if } x=1, u=-2$$

$$= \frac{-1}{2} (e^{-2} - 1)$$

$$14. \int x 3^{x^2} dx = \frac{1}{2} \int 3^u du$$

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= \frac{1}{2} \cdot \frac{3^{x^2}}{\ln 3} + C$$

$$= \frac{3^{x^2}}{\ln 9} + C$$

$$15. \int x^2 \sec^2 x^3 = \frac{1}{3} \int \sec^2 u du$$

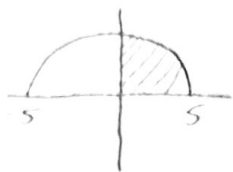
$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

$$= \frac{1}{3} \tan(x^3) + C$$

$$16. \int_0^5 \sqrt{25-x^2} dx = \frac{25\pi}{4}$$



$$17. \int_0^1 (x-x^2) dx = \left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3} \right) - (0)$$

$$= \frac{1}{6}$$

$$18. \int_0^{\frac{\pi}{2}} \cos x dx = \sin x \Big|_0^{\frac{\pi}{2}}$$

$$= \sin \frac{\pi}{2} - \sin 0$$

$$= 1 - 0$$

$$= 1$$

$$19. \int f'(x) dx = \int \cos x dx \quad f(0) = -5$$

$$f(x) = \sin x + C$$

$$-5 = C$$

$$f(x) = \sin x - 5$$

$$20. a) 30 \cdot 4 + 25 \cdot 4 + 16 \cdot 4 = \underline{284 \text{ gallons of oil}}$$

$$b) 25 \cdot 4 + 16 \cdot 4 + 7 \cdot 4 = \underline{192 \text{ gallons of oil}}$$

$$c) 27 \cdot 4 + 21 \cdot 4 + 13 \cdot 4 = \underline{256 \text{ gallons of oil}}$$