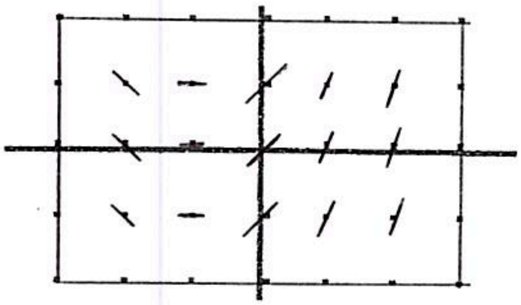
Name:

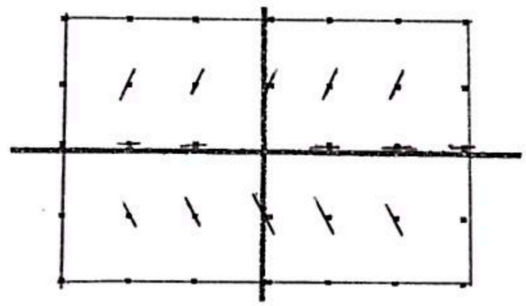
Key

Draw a slope field for each of the following differential equations. Each tick mark is one unit.

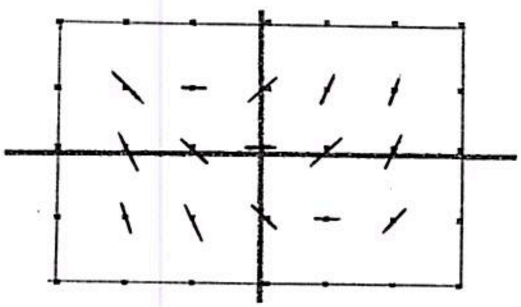
1. $\frac{dy}{dx} = x+1$ Separable!



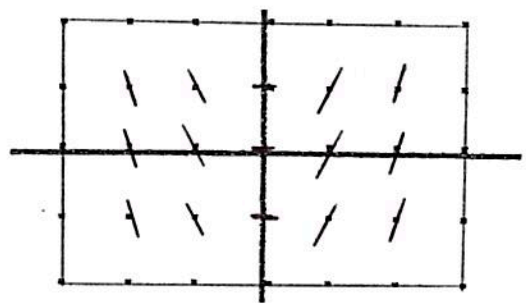
2. $\frac{dy}{dx} = 2y$ Separable!



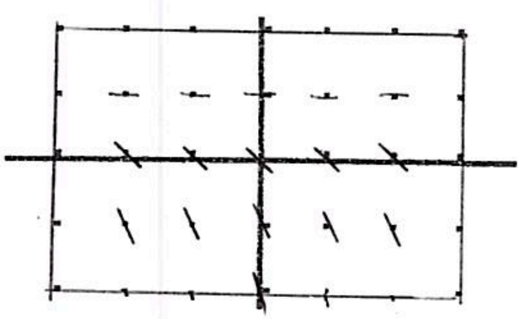
3. $\frac{dy}{dx} = x+y$ NOT SEPARABLE!



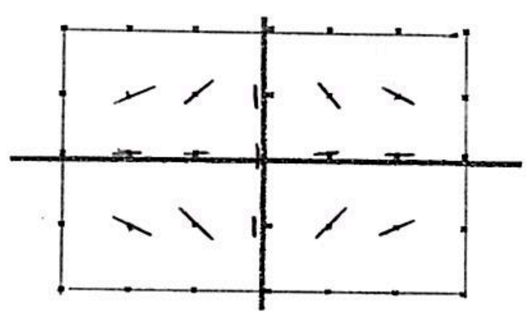
4. $\frac{dy}{dx} = 2x$ SEPARABLE!



5. $\frac{dy}{dx} = y-1$ SEPARABLE!

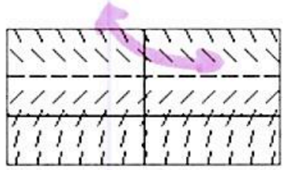


6. $\frac{dy}{dx} = -\frac{y}{x}$ SEPARABLE!

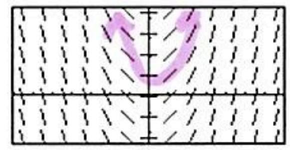


Match the slope fields with their differential equations.

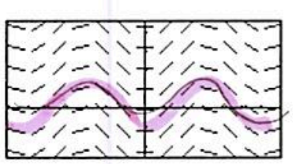
(A)



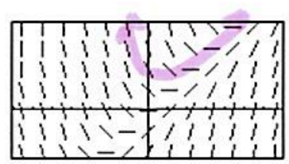
(B)



(C)



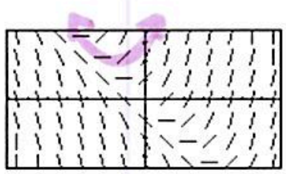
(D)



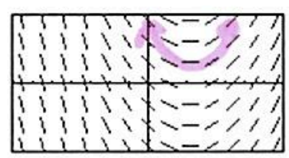
7. $\dfrac{dy}{dx} = \sin x$ C 8. $\dfrac{dy}{dx} = x - y$ D 9. $\dfrac{dy}{dx} = 2 - y$ A 10. $\dfrac{dy}{dx} = x$ B

Match the slope fields with their differential equations.

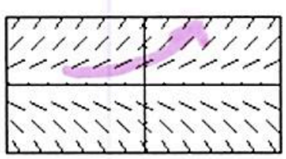
(A)



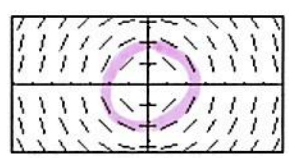
(B)



(C)

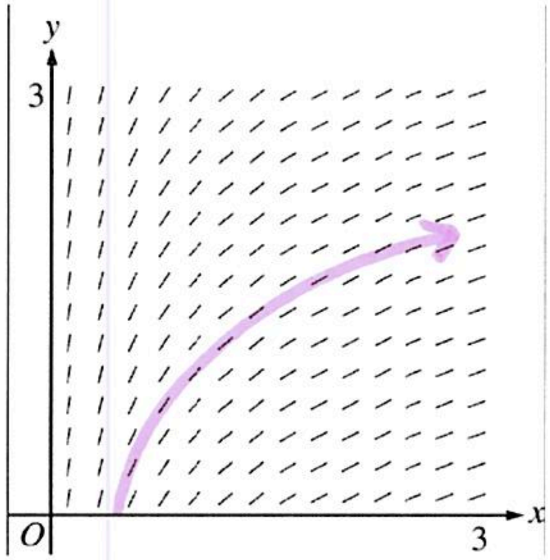


(D)



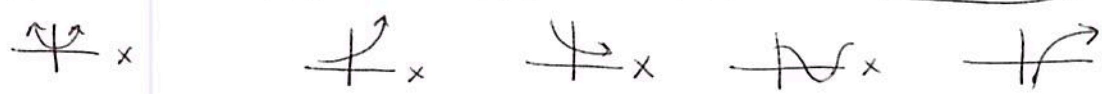
11. $\dfrac{dy}{dx} = 0.5x - 1$ B 12. $\dfrac{dy}{dx} = 0.5y$ C 13. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ D 14. $\dfrac{dy}{dx} = x + y$ A

From the May 2008 *AP Calculus Course Description:*
15.

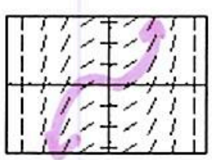


The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$



16.

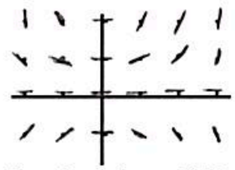


The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (C). Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

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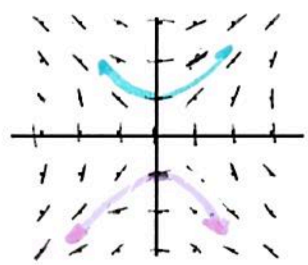
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(D) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(E) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

17. a) see paper

b) $\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x-1)$

$y - 1 = \frac{1}{2}(1.2 - 1)$

$y = 1.1$

$\therefore f(1.2) \approx 1.1$

c) $\int \frac{dy}{y} = \int \frac{x}{2}\,dx$

$\ln|y| = \frac{x^2}{4} + C \qquad (1,1)$

$\ln 1 = \frac{1}{4} + C$

$C = -\frac{1}{4}$

$\ln|y| = \frac{x^2}{4} - \frac{1}{4}$

$y = e^{\frac{x^2}{4} - \frac{1}{4}}$

$f(1.2) = e^{\frac{1.2^2 - 1}{4}} = 1.116$

d) $|\text{ERROR}| = |1.116 - 1.1| = .016$

$\therefore$ part c was an underestimate

$\frac{dy}{dx} = \frac{xy}{2}$

$\frac{d^2y}{dx^2} = \frac{1}{2}\left(x \cdot \frac{dy}{dx} + y \cdot 1\right)$

$\left.\frac{d^2y}{dx^2}\right|_{(1,1)} = \frac{1}{2}\left(1 \cdot \frac{1}{2} + 1\right) > 0$

$\therefore$ $f(x)$ is concave up @ $(1,1)$

because $f(x)$ is concave up, the tangent line approximation will be an underestimate.

18) $\frac{dy}{dx} = \frac{x}{y}$

a) see paper

b) see paper

c) $\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C \quad (0,1)$

$\frac{1}{2} = 0 + C$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{1}{2}$

$y^2 = x^2 + 1$

$y = \sqrt{x^2+1}$

d) see paper

e) $\frac{y^2}{2} = \frac{x^2}{2} + C \quad (0,-1)$

$\frac{1}{2} = C$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{1}{2}$

$y^2 = x^2 + 1$

$y = -\sqrt{x^2+1}$

Slope Fields Worksheet Solutions

7. C
8. D
9. A
10. B
11. B
12. C
13. D
14. A
15. E
16. D
17. (B) Tangent line: $y-1=\frac{1}{2}(x-1)$

$f(1.2)\approx 1.1$

(C) $y=e^{\frac{x^2-1}{4}}$
$f(1.2)=1.116$

(D) The estimate from part (b) was an underestimate. Since the graph of $y=e^{\frac{x^2-1}{4}}$ is concave up, the tangent line found in part (b) lies below the curve.

18. (C) $y=\sqrt{x^2+1}$
(E) $y=-\sqrt{x^2+1}$