## AP Calculus

$\qquad$

## First Semester Review 2015

1) If $f(x)=-x^{2}+4 x$, then the $\lim _{x \rightarrow 2} f(x)=$.
a) 12
b) none of these
c) 0
d) -12
e) 4
2) Find $\lim _{x \rightarrow 1} f(x)$ if $f(x)=\left\{\begin{array}{cc}3 x-2, & x \leq 1 \\ 2 x^{2}, & x>1\end{array}\right.$.
a) 1
b) 0
c) 2
d) The limit does not exist
e)none of these
3) Find $\lim _{x \rightarrow 1} f(x)$ for $\mathrm{f}(\mathrm{x})=\frac{x-1}{x^{2}-1}$.
a) 2
b) $\frac{1}{2}$
c) none of these
d) 0
e) The limit does not exist
4) Find the limit: $\lim _{x \rightarrow-1} \frac{x^{2}-5 x-6}{x+1}$.
a) none of these
b) $\infty$
c) -7
d) $-\infty$
e) 0
5) Find the limit: $\lim _{x \rightarrow 2^{-}} \frac{1}{x-2}$.
a) none of these
b) $-\infty$
c) $\infty$
d) 0
e) $-\frac{1}{4}$
6) Find all asymptotes of the graph of $g(x)=\frac{x+1}{x^{2}-1}$. Classify them as vertical or horizontal.
7) Find $\lim _{x \rightarrow \infty} \frac{2 x^{4}+6 x^{2}+5}{3+x^{4}}$.
a) $\frac{2}{3}$
b) $\infty$
c) 1
d) 2
e) None of these
8) Find $\mathrm{f}^{\prime}(\mathrm{x})$ if $f(x)=\frac{x^{2}-3 x}{x^{2}}$.
a) $1-\frac{3}{x}$
b) none of these
c) $\frac{3}{x^{2}}$
d) $\frac{2 x-3}{x^{2}}$
e) $\frac{2 x-3}{2 x}$
9) What are the 3 ways a derivative can fail to exist. Sketch a picture of each.
10) Find the derivative of $x^{-3}$.
a) $-\frac{3}{x^{2}}$
b) $\frac{1}{x^{4}}$
c) $-\frac{3}{x^{4}}$
d) -3
e) none of these
11) Find the instantaneous rate of change of w with respect to n if $w=\frac{9}{2 n}$.
12) Suppose the position equation for a moving object is given by $s(t)=3 t^{2}+2 t+5$, where $s$ is measured in meters and $t$ is measured in seconds. Find the velocity of the object when $t=2$. \{velocity is the derivative of position\}
a) $14 \mathrm{~m} / \mathrm{sec}$
b) $6 \mathrm{~m} / \mathrm{sec}$
c) none of these
d) $13 \mathrm{~m} / \mathrm{sec}$
e) $10 \mathrm{~m} / \mathrm{sec}$
13) Find the value of the derivative of $f(t)=\frac{t^{3}+2}{t}$ at the point $(-2,3)$.
a) $-\frac{11}{16}$
b) $-\frac{7}{2}$
c) $-\frac{9}{2}$
d) 12
e) none of these
14) If $f(x)=x^{\frac{3}{2}}$, then $f^{\prime}(4)=$
(A) -6
(B) -3
(C) 3
(D) 6
(E) 8
15) Find $\frac{d^{2} y}{d x^{2}}$ for $y=\frac{x+3}{x-1}$.
a) 0
b) $\frac{-8}{(x-1)^{3}}$
c) $\frac{-4}{(x-1)^{3}}$
d) $\frac{8}{(x-1)^{3}}$
e) none of these
16) Find $f^{\prime}(x)$ for $f(x)=\left(2 x^{2}+5\right)^{7}$.
a) $7\left(2 x^{2}+5\right)^{6}$
b) $7(4 x)^{6}$
c) none of these
d) $(4 x)^{7}$
e) $28 x\left(2 x^{2}+5\right)^{6}$
17) Find the $x$-values of all the points where the graph of the curve $y=24 x^{3}+6 x^{2}-12 x+6$ has a horizontal tangent.
a) $\frac{-1}{2}$ and $\frac{-1}{3}$
b) $\frac{-1}{2}$ and $\frac{1}{3}$
c) $\frac{1}{2}$ and $\frac{1}{3}$
d) $\frac{1}{2}$ and $\frac{-1}{3}$
e) none of these
18) If $f$ is a differentiable function, then $f^{\prime}(a)$ is given by which of the following?
I. $\quad \lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
II. $\lim _{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$
III. $\lim _{x \rightarrow a} \frac{f(x+h)-f(x)}{h}$
(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III
19) Find an equation of the tangent line to the graph of $x^{2}+3 y^{2}=4$ at the point $(1,1)$.
a) $y+1=-\frac{1}{3}(x+1)$
b) none of these
c) $x+3 y=2$
d) $y-1=-\frac{1}{3}(x-1)$
e) $y-1=-\frac{x}{3 y}(x-1)$
20) Find $\frac{d y}{d x}$ if $x y^{2}-y=x^{3}$.
a) $3 x^{2}-y^{2}$
b) none of these
c) $\frac{3 x^{2}-y^{2}}{2 x y-1}$
d) $\frac{3 x^{2}}{2 y-1}$
e) $y$
21) The top of a 25 -foot ladder is sliding down a vertical wall at a constant rate of 4 feet per minute. When the top of the ladder is 24 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?
22) Find the derivative of $y=\sec ^{2}(2 x)$.
23) The function $f$ is given by $f(x)=x^{4}+x^{2}-2$. On which of the following intervals is $f$ increasing?
a) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
c) $(0, \infty)$
d) $(-\infty, 0)$
e) $\left(-\infty,-\frac{1}{\sqrt{2}}\right)$
24) Find all intervals on which the function is concave upward: $f(x)=\frac{x^{2}+1}{x^{2}}$.
a) $(-\infty, \infty)$
b) $(-\infty,-1) \operatorname{and}(1, \infty)$
c) $(-\infty, 0) \operatorname{and}(0, \infty)$
d) $(1, \infty)$
e) None of these
25) State why Rolle's Theorem does not apply to the function $f(x)=\frac{2}{(x+1)^{2}}$ on the interval $[-2,0]$.
a) $f$ is not continuous on $[-2,0]$.
b) $f(-2) \neq f(0)$
c) $f$ is not differentiable at $x=-1$
d) Both a and c
e) None of these
26) If $f(x)=e^{3 \ln \left(x^{2}\right)}$, then $f^{\prime}(x)=$
(A) $e^{3 \ln \left(x^{2}\right)}$
(B) $\frac{3}{x^{2}} e^{3 \ln \left(x^{2}\right)}$
(C) $6(\ln x) e^{3 \ln \left(x^{2}\right)}$
(D) $5 x^{4}$
(E) $6 x^{5}$
27) The graph of $y=5 x^{4}-x^{5}$ has a point of inflection at
a) $(0,0)$ only
b) $(3,162)$ only
C) $(4,256)$ only
d) $(0,0)$ and $(3,162)$
e) $(0,0)$ and $(4,256)$
28) At what value of $x$ does $f(x)=\frac{x^{3}}{3}-x^{2}-3 x+5$ have a relative minimum?
a) - 1 only
b) 0 only
c) 1 only
d) 3 only
e) -1 and 3
29) Let $f(x)=2 x^{3}$. Find the value of $c$ that satisfies the Mean Value Theorem on the closed interval $[0,3]$
30). $\frac{d}{d x}\left(2^{3 x}\right)=$
(A) $2^{3 x-1}$
(B) $\left(2^{x-1}\right) 3$
(C) $3\left(2^{3 x}\right) \ln 2$
(D) $3\left(2^{x-1}\right) \ln 2$
(E) $\frac{3}{\mathrm{x} \ln 2}$
30) What does $f(x)=\left|x^{2}-3\right|$ look like? What about $(x)=\sqrt{|x+3|}$ ?
31) Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true?
a) $f$ is continuous at $x=0$
b) $f$ is differentiable at $x=0$
c) $f$ has an absolute minimum at $x=0$
d) Both a and c
e) Both b and c
32) If $f(x)=\ln \left(x+2+e^{-4 x}\right)$ find $\mathrm{f}^{\prime}(\mathrm{x})$.
33) If $y=\operatorname{Arcsin}\left(e^{2 x}\right)$, then $\frac{d y}{d x}=$
a) $\frac{2 e^{2 x}}{\sqrt{1-e^{4 x}}}$
b) $\frac{2 e^{2 x}}{1+e^{4 x}}$
c) $\frac{e^{2 x}}{1+e^{4 x}}$
d) $\frac{1}{\sqrt{1-e^{4 x}}}$
e) $\frac{1}{1+e^{4 x}}$
34) Find $y^{\prime}$ if $y=\frac{x^{2}}{2^{x}}$.
a) $\frac{x}{2^{x-2}}$
b) $\frac{2 x}{2^{x}(\ln 2)}$
c) $\frac{x(4-x)}{2^{x+1}}$
d) $\frac{x[2-x(\ln 2)]}{2^{x}}$
e) None of these
35) Let $f$ be the function defined by $f(x)=x^{3}+x$. If $g(x)=f^{-1}(x)$, what is the value of $g^{\prime}(2)$ if $g(2)=1$ ?
a) $\frac{1}{13}$
b) $\frac{1}{4}$
c) $\frac{7}{4}$
d) 4
e) 13
36) a) How do you find absolute extrema?
b) Local extrema?
c) Can a local extrema also be an absolute extrema?
d) Can a relative extrema also be a global extrema?
37) A farmer would like to enclose the garden adjacent to his house. He has 100 ft of fencing material. What would be the dimensions of the garden if he would like to maximize the area?
a) $50 \mathrm{ft} \times 50 \mathrm{ft}$
b) $25 \mathrm{ft} \times 50 \mathrm{ft}$
c) $30 \mathrm{ft} \times 40 \mathrm{ft}$
d) $40 \mathrm{ft} \times 60 \mathrm{ft}$
e) none of these

38) Of the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?
$V=\frac{4}{3} \pi r^{3}$
a) $40 \pi$
b) $80 \pi$
c) 800
d) $800 \mathrm{\pi}$
e) $3200 \pi$
39) The graph of $f(x)$ is shown at the right. Which of the following could be the graph of the derivative of $f(x)$.

a

b)

d)

c)

e)

41. A particle moves along the $x$-axis so that at time $t$ its position is given by $x(t)=t^{3}-6 t^{2}+9 t+11$.
a) What is the velocity of the particle at $t=0$ ?
b) During what time intervals is the particle moving to the left?
c) What is the total distance traveled by the particle from $t=0$ to $t=2$ ?
d) Is the speed of the particle increasing at $t=4$ ? Explain your answer.
