AP Calculus AB Name Key

Finding Derivatives using a Table

Two functions, f and g, are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are shown in the table.

x	0	1	2	3	4
f(x)	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
g(x)	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
f'(x)	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
g'(x)	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Based on the table, find the following derivatives:

1) $\frac{d}{dx}(f(x)+g(x))$, evaluated at x = 4.

$f'(4)+g'(4) = -\frac{4}{5} + -\frac{1}{3}$

$= \frac{-12 -5}{15}$

$= -\frac{17}{15}$

2) $\frac{d}{dx}(f(x)\cdot g(x))$, evaluated at x = 1.

$= f(1)\cdot g'(1) + g(1)\cdot f'(1)$

$= \frac{1}{3}\cdot\frac{2}{3} + 1\cdot\frac{5}{3}$

$= \frac{2}{9} + \frac{15}{9}$

$= \frac{17}{9}$

3) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$, evaluated at x = 0.

$= \frac{g(0)\cdot f'(0) - f(0)\cdot g'(0)}{[g'(0)]^2}$

$= \frac{-2\cdot\frac{3}{2} - \frac{1}{2}\cdot(-1)}{(-1)^2}$

$= -3 + \frac{1}{2}$

$= -\frac{5}{2}$

4) $\frac{d}{dx}(f(g(x)))$, evaluated at x = 3.

$= f'(g(3))\cdot g'(3)$

$= f'(2)\cdot g'(3)$

$= \frac{1}{4}\cdot(-3)$

$= -\frac{3}{4}$

Calculus AB
Review – Tangent Line Slope – Limit Definition

Name Key
Date

1. $f(x) = x^2 + 1$ at $x = 2$ a) Use the limit definition to find the derivative of f(x). $\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$ $\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{1}-\cancel{x^2}-\cancel{1}}{h}$ $\lim_{h\to 0} \frac{\cancel{h}(2x+\cancel{h})}{\cancel{h}} = 2x$	2. $f(x) = \frac{x}{x-2}$ at $x = 3$ a) Use the limit definition to find the derivative of f(x). $\lim_{h\to 0} \frac{\frac{x+h}{(x+h)-2} - \frac{x}{x-2}}{h}$ $\lim_{h\to 0} \frac{(x+h)(x-2) - x(x+h-2)}{(x-2)(x+h-2)} \cdot \frac{1}{h}$ $\lim_{h\to 0} \frac{\cancel{x^2}+\cancel{xh}-\cancel{2x}-2h-\cancel{x^2}-\cancel{xh}+\cancel{2x}}{(x-2)(x+h-2)} \cdot \frac{1}{h}$ $\lim_{h\to 0} \frac{-2}{(x-2)(x+h-2)} = \frac{-2}{(x-2)^2}$
b) Use the power rule to verify your answer. $f'(x) = 2x$	b) Use the quotient rule to verify your answer. $f'(x) = \frac{(x-2)\cdot 1 - x\cdot 1}{(x-2)^2}$ $f'(x) = \frac{-2}{(x-2)^2}$
c) Find the slope of the tangent line at x=2. $f'(2) = 4$	c) Find the slope of the tangent line at x=3. $f'(3) = \frac{-2}{(3-1)^2} = -2$
d) Write the equation of the tangent line at x=2. $y - 5 = 4(x-2)$ $f(2) = 4+1 = 5$ $f'(2) = 2(2) = 4$	d) Write the equation of the tangent line at x=3. $y - 3 = -2(x-3)$ $f(3) = \frac{3}{3-2} = 3$

1. $f' = 0$

2. $f' = -2$

3. $f' = -2$

4. $f' = -4x$

5. $f' = 8x^3 + 3x^2 - 2x$

6. $f = \frac{x^3}{3} + \frac{2}{3}$

$f' = x^2$

7. $f = \frac{1}{3}x^{-2}$

$f' = -\frac{2}{3}x^{-3} = \frac{-2}{3x^3}$

8. $f = 1 + x^{-1}$

$f' = \frac{-1}{x^2}$

9. $f' = (5x^2-3)(2x+1) + (x^2+x+4)(10x)$

$= \cancel{10x^3} - 6x + \cancel{5x^2} - 3 + \cancel{10x^3} + \cancel{10x^2} + 40x$

$= 20x^3 + 15x^2 + 34x - 3$

10. $f = 5x^{-5}$

$f' = \frac{-25}{x^6}$

11. $f = 5x^{-5} + 3x^{-2}$

$f' = \frac{-25}{x^6} - \frac{6}{x^3}$

12. $f = x^{\frac{1}{2}}$

$f' = \frac{1}{2\sqrt{x}}$

13. $f = x^{-1/2}$

$f' = -\frac{1}{2}x^{-3/2} = \frac{-1}{2\sqrt{x^3}}$

14. $f = x^{-3/2}$

$f' = -\frac{3}{2}x^{-5/2} = \frac{-3}{2\sqrt{x^5}}$

15. $f = x^{2/3} + x^{\frac{1}{2}}$

$f' = \frac{2}{3}x^{-1/3} + \frac{1}{2}x^{-1/2}$

$f' = \frac{2}{3\sqrt[3]{x}} + \frac{1}{2\sqrt{x}}$

16. $f' = 2\cos x + 3\sin x$

17. $f' = -4e^x + 5^x \ln 5$

18. $y' = (3x-1)\cdot 2 + (2x+4)\cdot 3$

$= 6x - 2 + 6x + 12$

$= 12x + 10$

19. $g' = \frac{(z^3-5)(2z) - (z^2+1)(3z^2)}{(z^3-5)^2}$

$= \frac{2z^3 - 10z - 3z^4 - 3z^2}{(z^3-5)^2}$

20. $h = (y^3+2y+1)^{-1}$

$h' = \frac{-(3y^2+2)}{(y^3+2y+1)^2}$

21. $f'(10) = g(10)\cdot h'(10) + h(10)\cdot g'(10)$

$= (-4)(35) + (560)(0)$

$= -140$

22. $y'(-3) = \dfrac{(1+9)\cdot z'(-3) - z(3)\,(2\cdot(-3))}{(1+(-3)^2)^2}$

$= \dfrac{10\cdot 15 - 6(-6)}{100}$

$= \dfrac{186}{100} = \dfrac{93}{50}$

23. $f' = 4(x^3-5x)^3(3x^2-5)$

$= 4(3x^2-5)(x^3-5x)^3$

24. $y' = -\sin(x^3)\cdot 3x^2$

$= -3x^2 \sin(x^3)$

25. $y = (\cos x)^3$

$y' = 3\cos^2 x \cdot (-\sin x)$

$y' = -3\sin x \cos^2 x$

26. $y' = -8(2x^2-6x+1)^{-9}(4x-6)$

$= \dfrac{-32x+48}{(2x^2-6x+1)^9}$

27. $y = (x^2-7x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x^2-7x)^{-1/2}\cdot(2x-7)$

$y' = \dfrac{2x-7}{2\sqrt{x^2-7x}}$

28. $y = (x^2-2x-5)^{-4}$

$y' = -4(x^2-2x-5)^{-5}(2x-2)$

$y' = \dfrac{-8x+8}{(x^2-2x-5)^5}$

29. $y = [\sin(\cos(4x))]^2$

S:	$[\quad]^2$	$2[\sin(\cos 4x)]$
C:	$\sin(\cos 4x)$	$\cos(\cos 4x)$
P:	$\cos(4x)$	$-\sin(4x)$
O:	$4x$	4

$y' = -8\sin(4x)\cos(\cos 4x)\sin(\cos(4x$

30.

$f(1) = 0$

$\lim_{x\to 1^-} f = 0$

$\lim_{x\to 1^+} f = 0$

$\therefore$ $f(x)$ is continuous @ $x=1$

$\lim_{x\to 1^-} f' = 2(1) = 2$

$\lim_{x\to 1^+} f' = 2 = 2$

$\therefore$ $f(x)$ is diff. @ $x=1$