CALCULUS AB
Review of INTEGRALS (Test #1)

Work the following on **notebook paper**

A. Evaluate the given integrals.

1. $\int\left(6x^2-5+\frac{7}{x^2}\right)dx$

2. $\int(3x+1)(2x-5)\,dx$

3. $\int\left(5+\sec x\tan x-\sec^2 x\right)dx$

4. $\int_0^{\pi/2} 2\cos x\,dx$

5. Given $f(x)=\begin{cases}2x, & x\le 1\\ 2, & x>1\end{cases}$. Evaluate: $\int_{1/2}^{5} f(x)\,dx.$

6. $\frac{d}{dx}\int_{x^2}^{2}\sqrt{t^2-t}\,dt$

B. Find the particular solution.

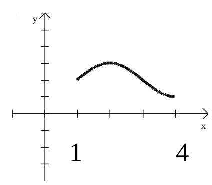
1. $y'=2+\frac{1}{x^2}$ and $y(1)=6$. Find $y(3)$.

2. If $f(3)=-4$, f' is continuous, and $\int_3^8 f'(x)\,dx=10$, find the value of $f(8)$.

C. Use the Fundamental Theorem of Calculus and the given graph.

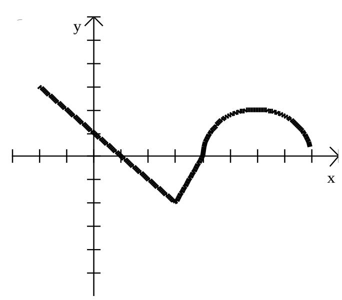
1. The graph of f' is shown on the right.

$\int_1^4 f'(x)\,dx=6.2$ and $f(1)=3$. Find $f(4)$.



2. The graph of f', consisting of two line segments and a semicircle, is shown on the right. Given that $f(-2)=5$, find:

(a) $f(1)$ (b) $f(4)$ (c) $f(8)$



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3. If $\int_2^5 (2g(x)+3)\,dx=17$, find $\int_2^5 g(x)\,dx.$

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4. Consider the function f that is continuous on the interval $[-5,5]$ and for which $\int_0^5 f(x)\,dx=4$. Evaluate:

(a) $\int_0^5 (f(x)+2)\,dx=$ (b) $\int_0^5 f(x)\,dx+2=$ (c) $\int_{-2}^{3} f(x+2)\,dx=$

5. The graph of a function f consists of a semicircle and two line segments as shown on the right.

Let $g(x)=\int_1^x f(t)\,dt$.

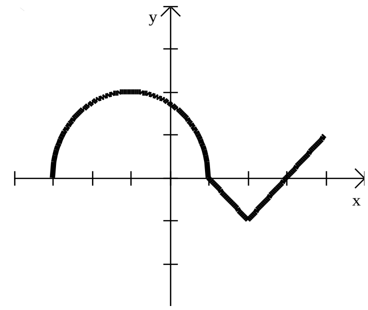
(a) Find $g(1)$, $g(3)$, $g(-1)$.

(b) On what interval(s) of x is g decreasing? Justify your answer.

(c) Find all values of x on the open interval $(-3,\ 4)$ at which g has a relative minimum. Justify your answer.

(d) Find the absolute maximum value of g on the interval $[-3,\ 4]$ and the value of x at which it occurs. Justify your answer.

(e) On what interval(s) of x is g concave up? Justify your answer.



Graph of f

(f) For what value(s) of x does the graph of g have an inflection point? Justify your answer.

(g) Write an equation for the line tangent to the graph of g at $x=-1$.

D. Reimann Sums

1. Given the values of the derivative $f'(x)$ in the table and that $f(0)=100$, estimate $f(6)$ Use a right Riemann sum.

x	0	2	4	6
$f'(x)$	10	18	23	25

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2. A bowl of soup is placed on the kitchen counter to cool. The temperature of the soup is given in the table below.

Time t (minutes)	0	5	8	11	12
Temperature $T(x)$ (°F)	105	99	97	94	93

(a) Find $\int_0^{12} T'(x)\,dx$. What does this value represent in the context of the problem?

(b) Find the average rate of change of $T(x)$ over the time interval t = 5 to t = 8 minutes.

(c) Estimate $T'(8)$. Show work and give units.

(d) What does $\frac{1}{11}\int_0^{11} T(x)\,dx$ mean in the context of this problem?

(e) Find $\frac{1}{11}\int_0^{11} T(x)\,dx$ using a left sum.

3 . The table below gives values of a continuous function. Use a midpoint Riemann sum with three equal subintervals to estimate the average value of f on [20, 50].

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

4.

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

E. Average Value

(a) Find the average value of f on the given interval.

(b) Find the value of c such that $f_{AVE} = f(c)$.

1. $f(x) = \sqrt{x}, \; [0, 4]$

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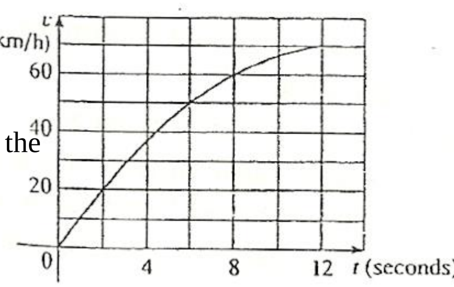
2. The velocity graph of an accelerating car is shown on the right.

(a) Estimate the average velocity of the car during the first 12 seconds by using a midpoint Riemann sum with three equal subintervals.

(b) At approximately what time was the instantaneous velocity equal to the average velocity?



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3. Suppose the $C(t)$ represents the daily cost of heating your house, measured in dollars per day, where t is time measured in days and $t = 0$ corresponds to January 1, 2010.. Interpret $\int_0^{90} C(t)\,dt$ and $\frac{1}{90-0}\int_0^{90} C(t)\,dt$.
