Review for Quiz (Related Rated, Optimization, and L'Hop)

Related Rates:

1. Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\text{cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 50cm?
2. A ladder 10m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6m from the wall?
3. A water tank has the shape of an inverted cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate if $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the tank is 3m deep.
4. Car A is traveling west at 50km/h and car B is traveling north at 60km/h. Both cars are headed for the intersection of the two roads. At what rate are the two cars approaching each other when car A is 300m and car B is 400m from the intersection?
5. Sven, who is 1.6 m tall, walks away from the base of a 4.5 m high lamppost at a rate of 1.2 m/s. At what rate is the length of his shadow increasing when he is 6m from the lamppost?
6. One leg of a right triangle is always 12 cm long, and the other leg is increasing at a rate of 1 cm/s. Find the rate of change of the hypotenuse (h), when it is 20 cm long.
7. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $1\text{cm}^3/\text{min}$. At what rate is the diameter decreasing when the diameter is 10cm?

Answers:

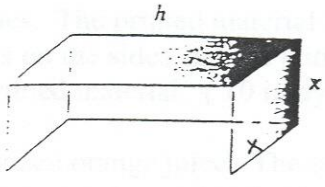
1. $\frac{1}{25\pi}$ cm/s
2. $\frac{-3}{4}$ m/s
3. 0.28 m/min
4. 78km/h
5. 0.662m/s
6. 0.8cm/s
7. $-\frac{1}{50\pi}$ cm/min

Optimization:

1. An open box is to be made from a 10 inch by 16 inch sheet of cardboard by cutting squares out of the four corners and folding up the sides. What dimensions of the box will yield the largest volume? What is the maximum volume of the box? (12 in by 6 in by 2 in , 144 cubic in)

2. An ecologist is conducting a research project on breeding pheasants in captivity. She must first construct suitable pens. She wants a rectangular region with two additional fences as shown in the diagram. Find the total maximum area that can be enclosed with 3000 meters of fencing.
   (w = 375m, l = 750m, area = 843,750 square meters)

3. A canvas wind shelter for the beach is to be constructed of 96 square feet of canvas that has been donated by a local company. The wind shelter is to have a back, two square sides and a top (see figure). What should be the dimensions of the shelter so that the space inside the shelter will be maximized? ( 4 ft by 8 ft )




4. A building supply store wants to build a rectangular outdoor storage facility on the rear wall of its building. An iron fence will enclose the facility on 3 sides and the wall of the building will be the fourth side. Find the dimensions of the storage facility with the largest area that can be built with 200 feet of fencing.
   ( 50 ft by 100 ft )

5. A rectangular cardboard poster is to contain 216 square inches of printed matter with 2 inch margins at the sides and 3 inch margins at the top and bottom. Find the dimensions of the poster using the least amount of cardboard. ( 16 in by 24 in )

6. Given the graph $y = (x-2)^2$, which point on the curve is closest to the point (-1,0)? (1,1)

L'Hop:

1.

Compute the following limits. Use l'Hôpital's Rule where appropriate but first check that no easier method will solve the problem.

(a) $\lim_{x\to 1} \dfrac{x^9 - 1}{x^5 - 1}$

(b) $\lim_{x\to 0} \dfrac{\sin(4x)}{\tan(5x)}$

(c) $\lim_{x\to 2} \dfrac{x^2 + x - 6}{x - 2}$

(d) $\lim_{x\to 1} \dfrac{x^2 + 2x - 2}{x^2 - 2x + 2}$

2. (a) $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

(b) $\lim_{x\to 0^+} \dfrac{\ln x}{1/x^2}$

(c) $\lim_{x\to\infty} \dfrac{\ln(x^4 - 2)}{\ln(3x^2 + 4)}$

Answers:

1. (a) 9/5 (b) 4/5 (c) 5 (d) 1

2. (a) 2 (b) 0 (c) 2