Related Rates

1. 

$\frac{dv}{dt} = 100 \frac{cm^3}{sec}$

$\left.\frac{dr}{dt}\right|_{d=50} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

when $d=50$, $r=25$

$100 = 4\pi(25)^2 \cdot \left.\frac{dr}{dt}\right|_{d=50}$

$\left.\frac{dr}{dt}\right|_{d=50} = \frac{1}{25\pi} \frac{cm}{sec}$

- The radius is growing by $\frac{1}{25}\frac{cm}{sec}$ when $d=50$.

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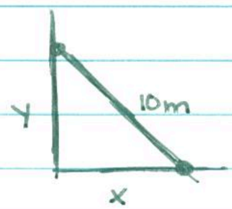
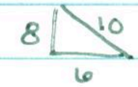
2. 



$x^2 + y^2 = 10^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = 0$

$6(1) + 8\left(\left.\frac{dy}{dt}\right|_{x=6}\right) = 0$

$\frac{dx}{dt} = 1 \frac{m}{sec}$

$\left.\frac{dy}{dt}\right|_{x=6} = ?$

$\left.\frac{dy}{dt}\right|_{x=6} = \frac{-3}{4}\frac{m}{sec}$

The ladder is sliding down the wall @ $\frac{3}{4}\frac{m}{sec}$ when $x=6$.

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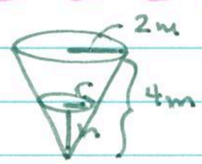
3. 

$\frac{r}{h} = \frac{2}{4}$

$r = \frac{1}{2}h$

$\frac{dv}{dt} = 2\frac{m^3}{min}$

$\left.\frac{dh}{dt}\right|_{h=3} = ?$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

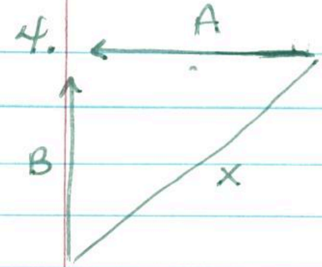
$\frac{dv}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$2 = \frac{1}{4}(\pi)(9)\left.\frac{dh}{dt}\right|_{h=3}$

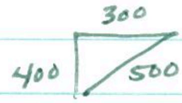
$\left.\frac{dh}{dt}\right|_{h=3} = .2829\ldots$

The depth of the water is increasing by .282 m/min when $h=3$.

4.



$$\frac{dA}{dt} = 50 \frac{km}{hr}$$



$$\frac{dB}{dt} = 60 \frac{km}{hr}$$

find $\left.\frac{dx}{dt}\right|_{A=300 + B=400}$

$$A^2 + B^2 = x^2$$

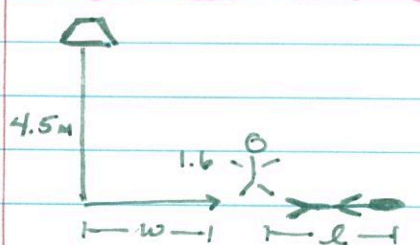
$$\cancel{2}A \frac{dA}{dt} + \cancel{2}B \frac{dB}{dt} = \cancel{2}x \frac{dx}{dt}$$

$$300(50) + 400(60) = 500\left(\frac{dx}{dt}\right)$$

$$\left.\frac{dx}{dt}\right|_{A=300} = 78 \text{ km/hr}$$

The two cars are approaching each other @ a rate of 78 km/hr when A = 300 and B = 400.

5.



$$\frac{4.5}{1.6} = \frac{l+w}{l}$$

$$4.5l = 1.6l + 1.6w$$

$$2.9l = 1.6w$$

$$\frac{dw}{dt} = 1.2 \frac{m}{s}$$

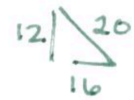
$$2.9 \frac{dl}{dt} = 1.6 \frac{dw}{dt}$$

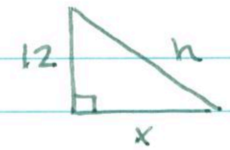
find $\left.\frac{dl}{dt}\right|_{w=6}$

$$2.9 \frac{dl}{dt} = 1.6(1.2)$$

$$\left.\frac{dl}{dt}\right|_{w=6} = 0.662\ldots \frac{m}{s}$$

The length of Sven's shadow is growing at a rate of 0.662 m/s when he is 6 m from the lamppost.



6. 

find $\left.\frac{dh}{dt}\right|_{h=20}$

$$\frac{dx}{dt} = 1 \ \frac{cm}{sec}$$

$$x^2 + 12^2 = h^2$$

$$\cancel{2}x \frac{dx}{dt} = \cancel{2}h \frac{dh}{dt}$$

$$16(1) = 20\left(\left.\frac{dh}{dt}\right|_{h=20}\right)$$

$$\left.\frac{dh}{dt}\right|_{h=20} = \frac{4}{5} \ \frac{cm}{sec}$$

The hypotenuse is increasing by $\frac{4}{5}$ cm/sec when $h = 20$.

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7. 

$$\frac{dV}{dt} = -1 \frac{cm^3}{min}$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$D = 2r$

$r = \frac{D}{2}$

$$\left.\frac{dD}{dt}\right|_{D=10} = ?$$

$$V = \frac{4}{3}\pi \cdot \frac{D^3}{8} = \frac{\pi}{6} D^3$$

$$\frac{dV}{dt} = \frac{1}{2}\pi D^2 \frac{dD}{dt}$$

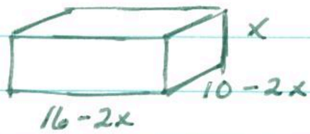
$$-1 = \frac{1}{2}\pi(100)\left.\frac{dD}{dt}\right|_{D=10}$$

$$\left.\frac{dD}{dt}\right|_{D=10} = \frac{-1}{50\pi} \ \frac{cm}{min}$$

When the diameter is 10 cm, the diameter is decreasing by $\frac{1}{50\pi} \frac{cm}{min}$.

Optimization

1.



$V = x(10-2x)(16-2x)$

$V = 4x^3 - 52x^2 + 160x$

$V' = 12x^2 - 54x + 160$

$V' = 0$ when $x = 2$

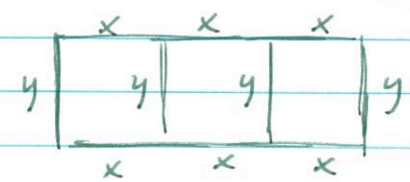
	$10x$	$-2x^2$
16	$160x$	$-32x^2$
$-2x$	$-20x^2$	$4x^3$

Dimensions of largest box:

2 in × 6 in × 12 in

Max Volume = 144 in^3

2.



$6x + 4y = 3000$

$6x = 3000 - 4y$

$x = \frac{3000 - 4y}{6}$

$x = 500 - \frac{2}{3}y$

$A = 3xy$

$A = 3(500 - \frac{2}{3}y) \cdot y$

$A = 1500y - 2y^2$

$A' = 1500 - 4y$

$A' = 0$ when $y = 375$

Max area when $y = 375$ m & $x = 250$ m

Total Area = 843,750 m^2

3.

$SA = 96 \ ft^3$

$SA = 2x^2 + 2xh$

$\frac{96 - 2x^2}{2x} = h$

* $V = x^2 \cdot h$

$V = x^{\cancel{2}}\left(\frac{48 - 1x^2}{\cancel{x}}\right) = 48x - x^3$

$V' = 48 - 3x^2$

$V' = 0$ when $x = 4$

Dimensions

$x = 4$ ft $h = 8$ ft

4.

$2y + x = 200$

$x = 200 - 2y$

$^{*}A = x \cdot y$

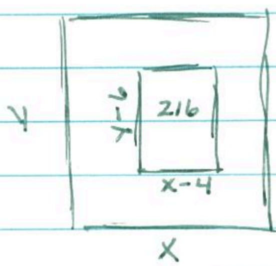
$A = (200 - 2y)\,y = 200y - 2y^2$

$A' = 200 - 4y$

$A' = 0$ when $y = 50$

Dimensions: 50 ft × 100 ft

5.



$(x-4)(y-6) = 216$

$y - 6 = \dfrac{216}{x-4}$

$y = \dfrac{216}{x-4} + 6$

$^{*}A = x \cdot y$

$A = x\left(\dfrac{216}{x-4} + 6\right)$

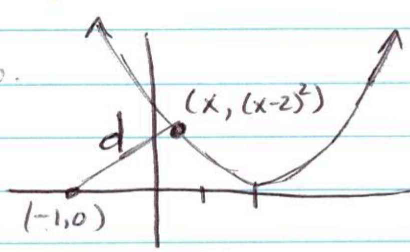
$A = \dfrac{216x}{x-4} + 6x$

$A' = \dfrac{(x-4)216 - 216x(1)}{(x-4)^2} + 6$

$A' = 0$ when $x = 16$

Dimensions: 16 in × 24 in

6.



$d = \sqrt{(x+1)^2 + (x-2)^4}$

$d' = \frac{1}{2}\left[(x+1)^2 + (x-2)^4\right]^{-\frac{1}{2}} \cdot \left[2(x+1) + 4(x-2)^3\right]$

(1,1)

$d' = 0$ when $x = \underline{\ \ 1\ \ }$

L'HOP

1. (a) $\lim_{x \to 1} \frac{x^9-1}{x^5-1}$

$\lim_{x \to 1} (x^9-1) = 0$

$\lim_{x \to 1} (x^5-1) = 0$

$\therefore$ L'HOP Applies

$$\lim_{x \to 1} \frac{x^9-1}{x^5-1} = \lim_{x \to 1} \frac{9x^8}{5x^4} = \lim_{x \to 1} \frac{9}{5}x^4 = \frac{9}{5}$$

(b) $\lim_{x \to 0} \frac{\sin 4x}{\tan 5x}$

$\lim_{x \to 0} \sin 4x = 0$

$\lim_{x \to 0} \tan 5x = 0$

$\therefore$ L'HOP Applies

$$\lim_{x \to 0} \frac{\sin 4x}{\tan 5x} = \lim_{x \to 0} \frac{4\cos 4x}{5\sec^2 5x} = \frac{4}{5}$$

(c) $\lim_{x \to 2} \frac{(x+3)\cancel{(x-2)}}{\cancel{x-2}} = 5$

(d) $\lim_{x \to 1} \frac{x^2+2x-2}{x^2-2x+2} = \frac{1}{1} = 1$

2. (a) $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

$\lim_{x \to 0} (e^x - e^{-x} - 2x) = 0$

$\lim_{x \to 0} (x - \sin x) = 0$

$= \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x}$

$\therefore$ L'HOP Applies

↳ continued on next page

2(a) cont.

$$\lim_{x \to 0} (e^x + e^{-x} - 2) = 0$$

$$\lim_{x \to 0} (1 - \cos x) = 0$$

$\therefore$ L'HOP Applies

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = \lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$$

$$\lim_{x \to 0} (e^x - e^{-x}) = 0$$

$$\lim_{x \to 0} \sin x = 0$$

$\therefore$ L'HOP APPLIES

$$\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x} = \lim_{x \to 0} \frac{e^x + e^{-x}}{\cos x} = \frac{2}{1} = 2$$

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2 (b) $$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}}$$

$$\lim_{x \to 0^+} \ln x = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$$

$\therefore$ L'HOP Applies

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$= \lim_{x \to 0^+} \frac{x^3}{-2x} = 0$$

2. (c) $\lim_{x\to\infty} \dfrac{\ln(x^4-2)}{\ln(3x^2+4)}$

$\lim_{x\to\infty} \ln(x^4-2) = \infty$

$\lim_{x\to\infty} \ln(3x^2+4) = \infty$

$\therefore$ L'HOP Applies

$= \lim_{x\to\infty} \dfrac{\frac{4x^3}{x^4-2}}{\frac{6x}{3x^2+4}}$

$= \lim_{x\to\infty} \dfrac{4x^3}{x^4-2} \cdot \dfrac{3x^2+4}{6x} = \lim_{x\to\infty} \dfrac{12x^5+16x^3}{6x^5-12x} = 2$