Notes on Product & Quotient Rule:

* Product Rule:

$$h(x) = f(x) \cdot g(x) \longrightarrow h'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

examples:

$y = x^2 \cdot \sin x$

$y' = x^2 \cdot \cos x + \sin x \cdot 2x$

or $\boxed{x(x\cos x + 2\sin x)}$

$g(x) = (3x+2)(5x^2+3x-4)$

$g'(x) = (3x+2)(10x+3) + (5x^2+3x-4) \cdot 3$

$= 30x^2 + 29x + 6 + 15x^2 + 9x - 12$

$= \boxed{45x^2 + 38x - 6}$

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* Quotient Rule:

$$h(x) = \frac{f(x)}{g(x)} \longrightarrow h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

examples:

$y = \dfrac{2x+1}{3x-4}$

$y' = \dfrac{(3x-4)\cdot 2 - (2x+1)\cdot 3}{(3x-4)^2}$

$y' = \dfrac{6x - 8 - 6x - 3}{(3x-4)^2}$

$\boxed{y' = \dfrac{-11}{(3x-4)^2}}$

$w(x) = \dfrac{\sin x}{x}$

$w' = \dfrac{x \cdot \cos x - \sin x \cdot 1}{x^2}$

$\boxed{w' = \dfrac{x\cos x - \sin x}{x^2}}$

More Trig:

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$y' = \frac{1}{\cos^2 x}$$

$$\boxed{y' = \sec^2 x}$$

$$y = \sec x = \frac{1}{\cos x}$$

$$y' = \frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$y' = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\boxed{y' = \sec x \tan x}$$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$y' = \frac{\sin x \cdot (-\sin x) - \cos x \cdot \cos x}{\sin^2 x}$$

$$y' = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$y' = \frac{-1}{\sin^2 x}$$

$$\boxed{y' = -\csc^2 x}$$

$$y = \csc x = \frac{1}{\sin x}$$

$$y' = \frac{\sin x \cdot 0 - 1(\cos x)}{\sin^2 x}$$

$$y' = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$\boxed{y' = -\csc x \cot x}$$

Higher Order Derivatives

$$f(x) \to f'(x) \to f''(x) \longrightarrow f'''(x) \longrightarrow f^{(4)}(x)$$

$$y \to \frac{dy}{dx} \to \frac{d^2y}{dx^2} \longrightarrow \frac{d^3y}{dx^3} \to \frac{d^4y}{dx^4}$$

position → velocity → acceleration → jerk → snap

Example:

A partical is moving along the x-axis such that its position is given by $x(t) = t^3 + 2t - 1$.

a) what is its velocity @ $t = 3$?

$v(t) = x'(t) = 3t^2 + 2t$

$v(3) = 27 + 6 = 33$

b) what is its acceleration @ $t = 3$?

$a(t) = x''(t) = 6t + 2$

$a(3) = 18 + 2 = 20$