Practice Problems

Name: Key

Identify all critical numbers and locate the absolute extrema of the function on the closed interval. Show all the Calculus that leads to your conclusion, the derivative, the location of critical numbers and the value of each extrema.

1. $f(x) = x^2 + 2x - 4\ [-2,1]$

$f'(x) = 2x + 2$

$0 = 2x + 2$

$-2 = 2x$

$-1 = x$

CN: $x = -1$

x	f(x)
-2	-4
-1	-5
1	-1

Abs. max of -1 @ x=1

Abs. min of -5 @ x=-1

2. $f(x) = \cos 2x\ [0, \pi]$ ↗ chain Rule

$f'(x) = -\sin 2x(2) = -2\sin 2x$

$0 = -2\sin 2x$

$2x = \pi \quad 2x = 2\pi$

$x = \frac{\pi}{2} \quad x = \pi$

CN: $x = \frac{\pi}{2}, \pi$

x	f(x)
0	1
$\frac{\pi}{2}$	-1
π	1

Abs max of 1 @ $x = 0, \pi$

Abs min of -1 @ $x = \frac{\pi}{2}$

3. Let f be a continuous function on [-4, 12]. If f(-4)=-2 and f(12)=6, then the Mean Value Theorem guarantees that

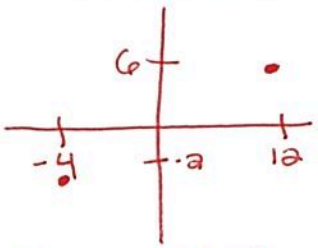
~~A.~~ f(4) = 2.

~~B.~~ $f'(4) = \frac{1}{2}$ can't prove 4

C. $f'(c) = \frac{1}{2}$ for at least one c between -4 and 12. ✓

~~D.~~ $f(c) = 0$ for at least one c between -4 and 12. IVT

~~E.~~ f(4) = 0.



AROC: $\frac{f(12) - f(-4)}{12 + 4} = \frac{6+2}{16}$

$= \frac{1}{2}$

4. The value of c that satisfies the Mean Value Theorem for Derivatives on the interval [0,5] for the function $f(x) = x^3 - 6x$ is

AROC: $\frac{f(5)-f(0)}{5-0} = \frac{95-0}{5} = 19$

A. $-\frac{5}{\sqrt{3}}$ B. 0 C. 1

D. $\frac{5}{3}$ (E.) $\frac{5}{\sqrt{3}}$

$f'(x) = 3x^2 - 6$

$3x^2 - 6 = 19$

$\frac{3x^2}{3} = \frac{25}{3} = \pm\frac{5}{\sqrt{3}}$ only + only interval

5. Determine the value for c on [2,5] that satisfies the Mean Value Theorem for $f(x) = \frac{x^2-3}{x-1}$

AROC: $\frac{f(5)-f(2)}{5-2} = \frac{(5.5)-1}{3} = \frac{3}{2}$

A. -1 B. 2 (C.) 3

D. 4 E. 5

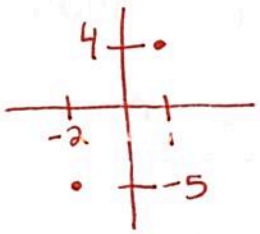
$f'(x) = \frac{(x-1)(2x)-(x^2-3)(1)}{(x-1)^2} = \frac{2x^2-2x-x^2+3}{(x-1)^2} = \frac{x^2-2x+3}{(x-1)^2}$

$\frac{3}{2} = \frac{x^2-2x+3}{x^2-2x+1}$

$3x^2-6x+3 = 2x^2-4x+6 \rightarrow x^2-2x-3=0$

$(x-3)(x+1)$ $x = 3, -1$

6. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?



(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$. ✓IVT

(B) There exists c, where $-2 < c < 1$, such that $f(c) = 3$. ✓ IVT

(C) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

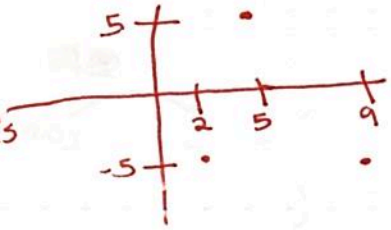
(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$. ✓MVT

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$. ✓ EVT

$\frac{f(1)-f(-2)}{1-(-2)} = \frac{4+5}{3}$

AROC: $= \frac{9}{3} = 3$

7. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?



I. f has at least 2 zeros. ✓ IVT

II. The graph of f has at least one horizontal tangent. ✓Rolle's

III. For some c, $2 < c < 5$, $f(c) = 3$. ✓ IVT

(A) None (B) I only (C) I and II only

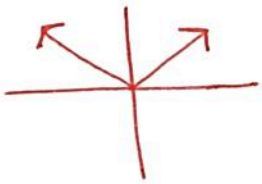
(D) I and III only (E) I, II and III

8. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$. ✓

II. f is differentiable at $x = 0$. X sharp turn

III. f has an absolute minimum at $x = 0$. ✓



(A) I only (B) II only (C) III only

(D) I and III only (E) II and III only

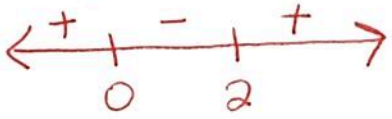
9. $f(x) = x^3 - 3x^2$

Sketch a graph of f(x) labeling all relative extrema and points of inflection.

$f'(x) = 3x^2 - 6x$

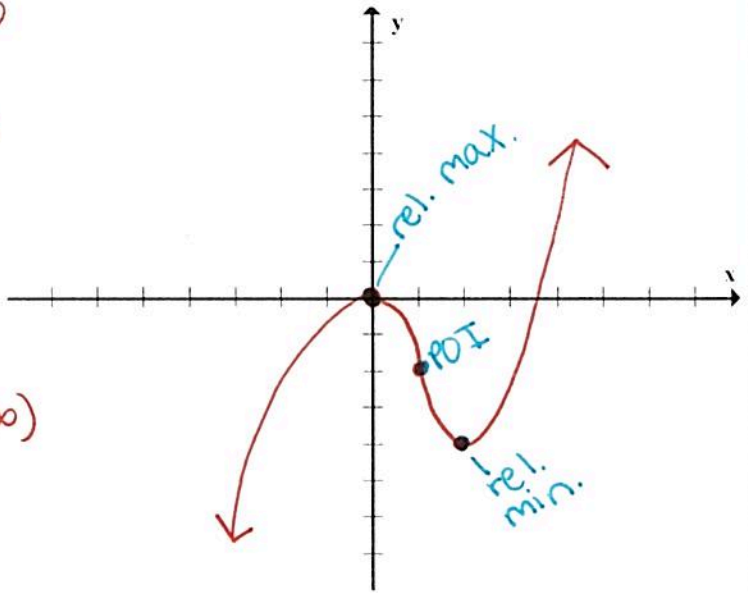
$0 = 3x(x-2)$

$x = 0, 2$



Rel. max 0 @ x=0

Rel min -4 @ x=2



Critical Values: $x = 0, 2$

Interval(s) of increase: $(-\infty, 0] \cup [2, \infty)$

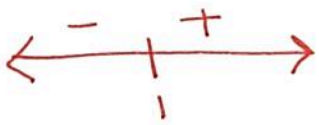
Interval(s) of decrease: $[0, 2]$

$f''(x) = 6x - 6$

$0 = 6x - 6$

$6 = 6x$

$x = 1$



x	f(x)
0	0
1	-2
2	-4

PPOIs: $x = 1$ POI: $(1, -2)$

Interval(s) of concave up: $[1, \infty)$

Interval(s) of concave down: $(-\infty, 1]$