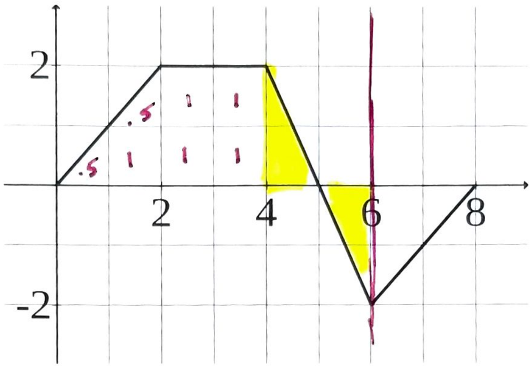
Calculus AB
FTC 2 & Integral Review
Non Calculator

Name ______________________________

1. Let f be the piecewise linear function graphed below, for $0 \le x \le 8$.

What is the value of $\int_0^6 f(x)\,dx$?

A) 6
B) 4
C) 13
D) 8
E) 1



Use this graph for Problems 1-3.

2. Using the above graph, what is $\int_0^6 [f(x)+1]\,dx$?

$\int_0^6 f(x)\,dx + \int_0^6 1\,dx = 6 + 1(6) = 12$

3. If $g(x) = \int_3^x f(t)\,dt$, find

A) g(5)

$g(5) = \int_3^5 f(x)\,dx = 3.5$

B) g′(5)

$g'(5) = f(5) = 0$

C) g″(5)

$g''(5) = f'(5) = -2$

Evaluate the Definite Integral. You must show all steps.

4. $\int_{-1}^{2} (3t^2 - 1)\,dt$

$\left(\frac{3t^3}{3} - t\right)\Big|_{-1}^{2}$

$= (8-2) - (-1+1)$

$= 6$

5. $\int_{-2}^{1} 6x\,dx$

$3\frac{6x^2}{2}\Big|_{-2}^{1} = 3 - 3(4)$

$= -9$

6. $\int_1^4 \left(-\frac{4}{x^2}+2\right)dx$

$\int_1^4 -4x^{-2}\,dx + \int_1^4 2\,dx$

$\frac{+4x^{-1}}{+1}\Big|_1^4 + 2x\Big|_1^4$

$\left(\frac{4}{4}-\frac{4}{1}\right)+(8-2)$

$-3+6 = 3$

7. $\int_1^9 \frac{x^2+2\sqrt{x}}{x}dx$

$\int_1^9 (x+2x^{-1/2})\,dx$

$\left(\frac{x^2}{2}+2x^{1/2}\cdot 2\right)\Big|_1^9$

$\left(\frac{81}{2}+4(3)\right)-\left(\frac{1}{2}+4\right)$

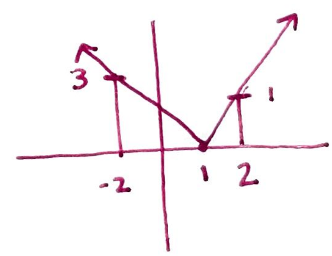
$40+12-4=48$

8. $\int_0^{\frac{\pi}{3}} \sec^2 x\,dx$

$\tan x \Big|_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan 0$

$= \sqrt{3}$

9. $\int_{-2}^{2} |x-1|\,dx$

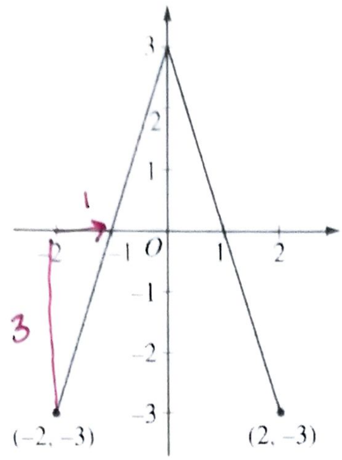


$\frac{9}{2}+\frac{1}{2} = 5$

10. Use the Second Fundamental Theorem of Calculus to find F '(x).

$F(x)=\int_{x^2}^{-2}\frac{t}{t^2+1}dt = \left(\frac{-2}{4+1}\right)\cdot 0 - \left(\frac{x^2}{x^4+1}\right)\cdot 2x$

$= \frac{-2x^3}{x^4+1}$



Graph of f

The graph of the function f shown above consists of two lines segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

$g(-1) = \int_0^{-1} f(t)\,dt$

$= -\left(\frac{-1 \cdot 3}{2}\right) = 1.5$

$g'(-1) = f(-1)$

$= 0$

$g''(-1) = f'(-1)$

$= 3$

(b) For what values of x in the open interval $(-2,2)$ is g increasing? Explain your reasoning.

g is increasing when $g' > 0$

$g' = f$

$f > 0$ on $(-1, 1)$ $\therefore$ g is increasing on $(-1,1)$

(c) For what values of x in the open interval $(-2,2)$ is the graph of g concave down? Explain your reasoning.

g is cc↓ when $g'' < 0$

g'' is the slope of f

f is decreasing on $(0, 2)$ $\therefore$ g is cc↓ on $(0,2)$

(d) Write the equation of the line tangent to g(x) at x = -1.

$y - g(-1) = g'(-1)(x+1)$

$y - 1.5 = 0(x+1)$

$y = 1.5$