Key - 2017

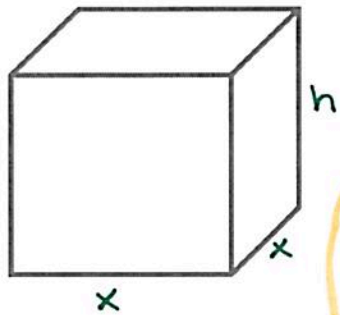
Optimization Notes

Calculator Active

1. A box with a square base and no top is to hold 32 cubic inches. Find the dimensions that require the least building material.

Volume

↳ Surface Area



$32 = x^2 \cdot h$

$h = \frac{32}{x^2}$

* $S = x^2 + 4xh$

$S = x^2 + 4x\left(\frac{32}{x^2}\right)$

$S = x^2 + \frac{128}{x}$

use calculator $Y_1 = S'$ $Y_2 = 0$

$S' = 2x - \frac{128}{x^2}$

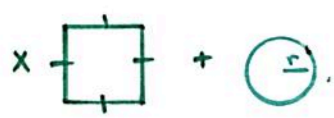
$S' = 0$ when $x = 4$

− + (4)

Rel Min

The dimensions that require the least building material are 4in x 4in x 2in.

2. One 4 foot piece of wire is used to form a square and a circle. How much of the wire should be used for the square and how much for the circle to enclose a minimum total area.



$4x + 2\pi r = 4$

$4x = 4 - 2\pi r$

$x = 1 - \frac{\pi}{2} r$

* $A = x^2 + \pi r^2$

$A = \left(1 - \frac{\pi}{2} r\right)^2 + \pi r^2$

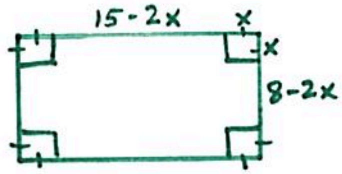
$A' = 2\left(1 - \frac{\pi}{2} r\right) \cdot \left(-\frac{\pi}{2}\right) + 2\pi r$ ← use calculator $A' = Y_1$ $0 = Y_2$

$A' = 0$ when $r = .28004...$

− + (.2800...)

Rel Min

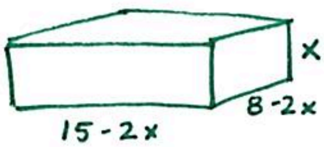
1.759 ft. should be used for the circle and 2.240 ft should be used for the square.



The maximum volume of the box that can be made is $90.740\ in^3$.

1979 AB 3 and BC 3

Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.



$V = (15-2x)(8-2x)(x)$

$V = (15-2x)(8x-2x^2)$

$V' = (15-2x)(8-4x) + (8x-2x^2)(-2)$

$V' = 0$ when $x = 1^2/_3$ or $x = 6$

$+ \quad - \quad +$; $1\frac{2}{3}$; 6

Rel max

1971 AB 4 and BC 1

Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$.

$A = (f-g) \cdot 2x$

$A = [(18-x^2) - (2x^2-9)] \cdot 2x$

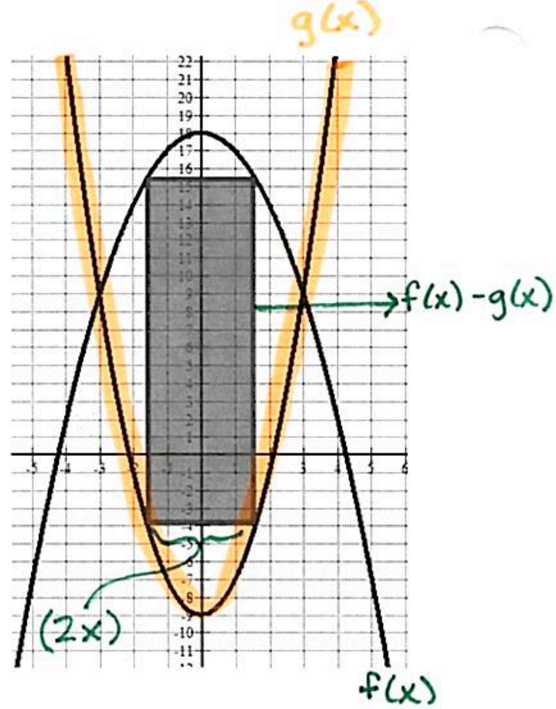
$A = (27 - 3x^2) \cdot 2x$

$A = 54x - 6x^3$

$A' = 54 - 18x^2 \qquad 18(3-x^2)$

$A' = 0$ when $x = \sqrt{3}$; $+ \quad -$; $\sqrt{3}$

Rel max



Maximum area = 62.353