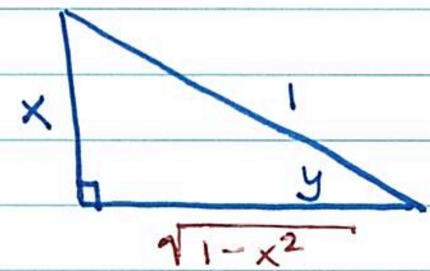
Notes on the Derivative of Inverse Trig Functions

$y = \arcsin x$

$\sin y = \frac{x}{1}$ O / H



now, since $y = \sin x$ and $y = \arcsin x$ are inverses of each other, we can use $y = \sin x$ to help us find the derivative of $\arcsin x$.

$f(x) = \sin x$
$f'(x) = \cos x$
$f'(y) = \frac{\sqrt{1-x^2}}{1} \left(\frac{A}{H}\right)$

all we need to do is take the reciprocal

$y = \arcsin x$
$y' = \frac{1}{\sqrt{1-x^2}}$ } specific

$y = \arcsin u$
$y' = \frac{du}{\sqrt{1-u^2}}$ } general

example:

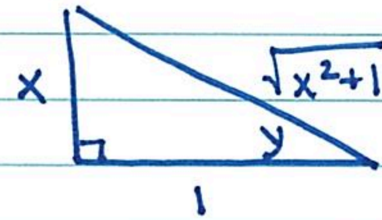
$u = x^2+3$
$du = 2x$

$y = \arcsin(x^2+3)$

$y' = \frac{2x}{\sqrt{1-(x^2+3)^2}}$

$y = \arctan x$

$\frac{O}{A}$ $\frac{x}{1} = \tan y$



$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f'(y) = \left(\frac{H}{A}\frac{\sqrt{x^2+1}}{1}\right)^2 = \frac{x^2+1}{1}$

$y = \arctan x$
$y' = \dfrac{1}{x^2+1}$ } specific

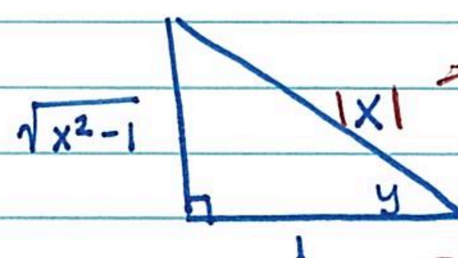
$y = \arctan u$
$y' = \dfrac{du}{u^2+1}$ } general

example:
$y = \tan^{-1}(3x-1)$ $\quad u = 3x-1$, $du = 3$

$y' = \dfrac{3}{(3x-1)^2+1}$

$y = \text{arc}\sec x$

$\frac{H}{A}$ $\frac{x}{1} = \sec y$



→ hypotenuse has to be positive

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f'(y) = \frac{|x|}{1} \cdot \frac{\sqrt{x^2-1}}{1} = \frac{|x|\sqrt{x^2-1}}{1}$

$y = \text{arcsec}\, x$
$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$ } general

$y = \text{arcsec}\, u$
$y' = \dfrac{1}{|u|\sqrt{u^2-1}}$ } specific

example: $y = \text{arcsec}\, x^2$ $\quad u = x^2$

$y' = \dfrac{2x}{x^2\sqrt{x^4-1}}$ $\quad du = 2x$

don't need ←

A few more formulas: (they can be proved the same way)

$$y = \operatorname{arc}\cos u \qquad y' = \frac{-du}{\sqrt{1-u^2}}$$

$$y = \operatorname{arc}\cot u \qquad y' = \frac{-du}{u^2+1}$$

$$y = \operatorname{arc}\csc u \qquad y' = \frac{-du}{|u|\sqrt{u^2-1}}$$

A few more examples:

★ $y = 5\csc^{-1}(x^2+1)$ $\quad u = x^2+1$

$du = 2x$

$$y' = \frac{-5\cdot 2x}{|x^2+1|\sqrt{(x^2+1)^2-1}}$$

$$y' = \frac{-10x}{(x^2+1)\sqrt{x^4+2x^2}}$$

chain rule:

s: $\cos(\ )$	$-\sin(\ )$
c: $\arcsin 3x$	$\frac{3}{\sqrt{1-9x^2}}$

★ $y = \cos(\arcsin(3x))$

$$y' = -\sin(\arcsin(3x)) \cdot \frac{3}{\sqrt{1-9x^2}}$$

$$y' = \frac{-3x\cdot 3}{\sqrt{1-9x^2}}$$

$$y' = \frac{-9x}{\sqrt{1-9x^2}}$$