Key - 2017

# Notes on EVT, MVT, and Rolle's Theorem

Review:

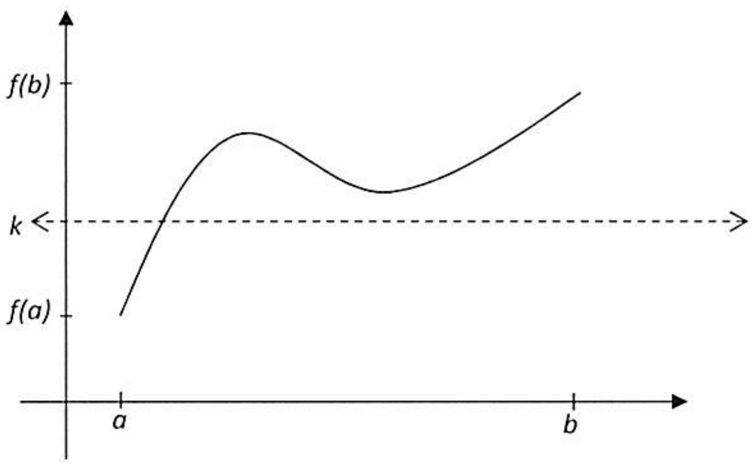
Intermediate Value Theorem:

Must be true:

1. f(x) must be cont. on [a, b]
2. $f(a) \le K \le f(b)$

Conclusion:

There must exist a c on [a, b] such that f(c)=K.



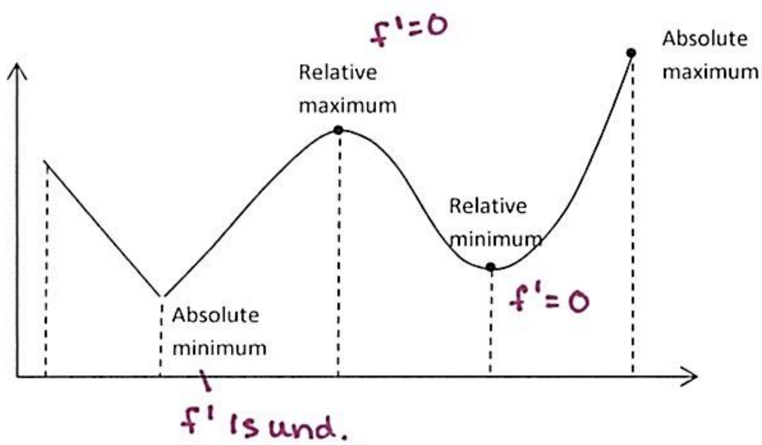

Limit Definition of a Derivative:

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} = f'(c)$$

$$f'(c) = \lim_{h \to 0} \frac{f(c+x)-f(c)}{h}$$

Absolute vs. Relative (local) Extrema




*Endpoints can never by relative.

Absolute Extrema:

- The highest and lowest values on a function.
- can occur at endpts or critical #s

Relative Extrema:

- The turning pts of a function.
- can occur only at critical #s.

Critical Values:

- Possible extrema
- where f' = 0 or f' is undefined.

Extreme Value Theorem:

Must be true:

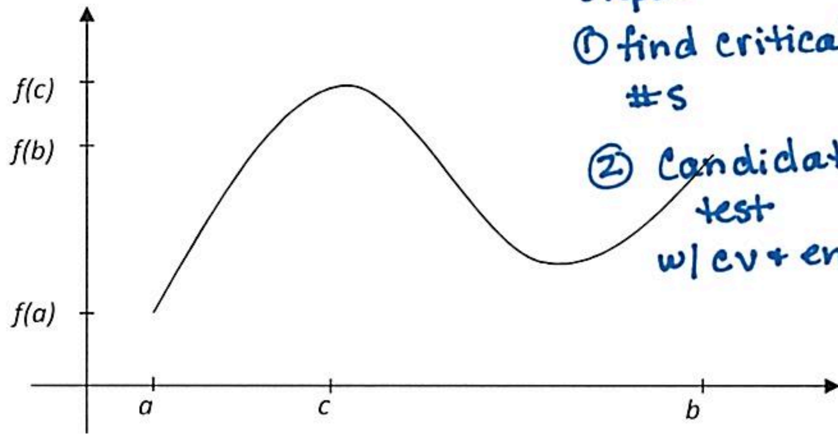
1. $f(x)$ must be cont. on $[a,b]$

Conclusion:

There must be an

abs. min & an abs. max.

$f(a) \le f(x) \le f(c)$




Steps:
① find critical #s
② Candidates test w/ cv + endpts.

Examples:

1) Find the absolute extrema for f(x) $=3x^4 - 4x^3$ on the interval [-1,2]

① $f' = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

CN: 0, 1

| x | f(x) |
|---|---|
| -1 | 7 |
| 0 | 0 |
| 1 | -1 |
| 2 | 16 |

Abs. max is 16 and occurs @ x=2.

Abs. min is -1 and occurs @ x=1.

3) Find the absolute extrema for $f(x) = 2x - 3x^{\frac{2}{3}}$ on the interval [-1,3]

$f' = 2 - 2x^{-1/3}$

$= 2 - \frac{2}{\sqrt[3]{x}}$

$= \frac{2\sqrt[3]{x} - 2}{\sqrt[3]{x}} = 0$

$\sqrt[3]{x} = 0$

CN: 1, 0

| x | f(x) |
|---|---|
| -1 | -5 |
| 0 | 0 |
| 1 | -1 |
| 3 | $6-3\sqrt[3]{9}$ $\approx -.240$ |

Abs. max = 0 @ x=0

Abs. min = -5 @ x=-1

2) Find the absolute extrema of $f(x) = 2x^3 + 3x^2 - 12x$ on the interval: [-4. 2]

$f' = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

CN: -2, 1

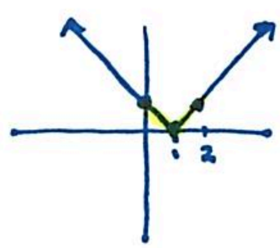
| x | f(x) |
|---|---|
| -4 | -32 |
| -2 | 20 |
| 1 | -7 |
| 2 | 4 |

Abs. max is 20 @ x = -2

Abs. min is -32 @ x = -4

4) Find the absolute extrema for f(x) = |x-1| on the interval [0,2]

CN @ x=1




| x | f(x) |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 2 | 1 |

Abs. Max is 1 @ x=0,2

Abs. Min is 0 @ x=1

Mean Value Theorem:

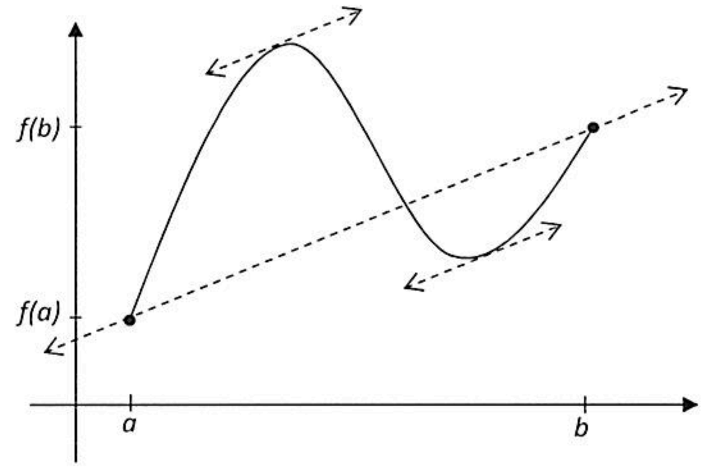
Must be true:

1. f(x) is cont. on [a,b]

2. f(x) is diff on (a,b)

Conclusion

$f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c on (a, b)




Rolle's Theorem(a special case of MVT):

Must be true:

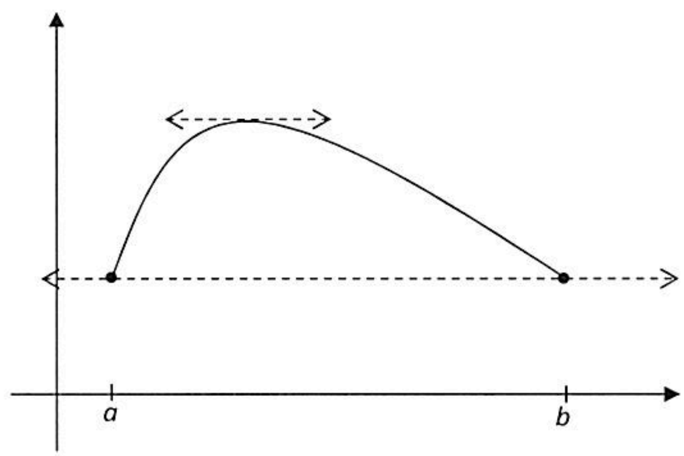
1. f(x) is cont. on [a,b]

2. f(x) is diff. on (a,b)

3. f(a) = f(b)

Conclusion

f'(c)=0 for some c on (a,b)




Examples:

1. Let $f(x)$ be the function given by $f(x) = x^3 - 7x + 6$. Find the number c that satisfies the conclusion of the Mean Value Theorem for $f(x)$ on [1,3].

$f'(x) = 3x^2 - 7$

$f(3) = 12$

$f(1) = 0$

$AROC = \frac{12-0}{2} = 6$

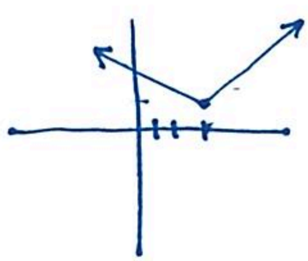
$c = \sqrt{\frac{13}{3}}$

$3x^2 - 7 = 6$

$3x^2 = 13$

$\sqrt{x^2} = \pm\sqrt{\frac{13}{3}}$ → $-\sqrt{\frac{13}{3}}$ is not on the given interval.

2. $f(x) = |x-3| + 1$ Can the Mean Value Theorem be used to show that there is an x-value for $f(x)$ on [1,4] where the slope of the tangent line is equal to the slope of the tangent line? Justify your conclusion.



f(x) is cont. on [1,4]

f(x) is NOT diff. @ x = 3

$\lim_{x\to3^-} f'(x) = \lim_{x\to3^+} f'(x)$

∴ MVT cannot be used.

Determine whether Rolle's Theorem can be applied. If so, find $c$. If not, tell why.

1. $f(x) = x^4 - 2x^2$ on [-2, 2].

$f(x)$ is a polynomial, therefore it is cont. and differentiable for all $x \in \mathbb{R}$.

$f(2) = 8$

$f(-2) = 8$

$\therefore$ Rolle's Applies

$f' = 4x^3 - 4x$

$0 = 4x^3 - 4x$

$0 = 4x(x-1)(x+1)$

$c = 0, 1, -1$

**2. Let $f$ be a function that is differentiable on the interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

T I. $f$ has at least 2 zeros. IVT

T II. The graph of $f$ has at least one horizontal tangent. Rolles

T III. For some $c$, $2 < c < 5$, $f(c) = 3$. IVT

F **IV. $f'(x) = 3$ for some c, 2 < c < 5.** MVT

$$\frac{f(5) - f(2)}{5 - 2} = \frac{10}{3} \neq 3$$