Key

Related Rates Notes – Day 2

1. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

$\frac{dr}{dt} = .04 \text{ cm/sec}$

(Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

$\left.\frac{dV}{dt}\right|_{r=10} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{r=10} = 4\pi(100).04$

$\left.\frac{dV}{dt}\right|_{r=10} = 16\pi \text{ cm}^3/\text{sec}$

when $r = 10$ cm, the Volume is increasing at a rate of $16\pi \text{ cm}^3/\text{sec}$

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

$\left.\frac{dA}{dt}\right|_{V=36\pi} = ?$

$36\pi = \pi r^2$ when $r = 6$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=6} = 2\pi(6) \cdot .04$

$\left.\frac{dA}{dt}\right|_{V=36\pi} = 1.507 \text{ cm}^2/\text{sec}$

when the volume of the sphere is $36\pi \text{ cm}^3$, the area of the cross section through the center is increasing at a rate of $1.507 \text{ cm}^2/\text{sec}$.

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

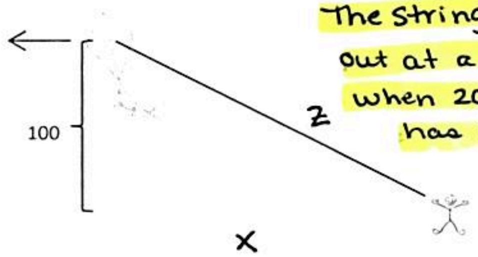
$\frac{dV}{dt} = \frac{dr}{dt}$ when $r = \sqrt{\frac{1}{4\pi}}$ cm

$4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}$

$4\pi r^2 = 1$

$r = \sqrt{1/4\pi}$

2. A kite 100 feet above the ground is being blown away from the person holding the string in a direction parallel to the ground at the rate of 10 feet per second. At what rate must the string be let out when the length of the string already let out is 200 ft?



The string is being let out at a rate of $5\sqrt{3}$ ft/sec when 200ft of string has been let out.

$100^2 + x^2 = z^2$

$\cancel{2}x \frac{dx}{dt} = \cancel{2}z \frac{dz}{dt}$

$100\sqrt{3} \cdot 10 = 200 \cdot \left.\frac{dz}{dt}\right|_{z=200}$

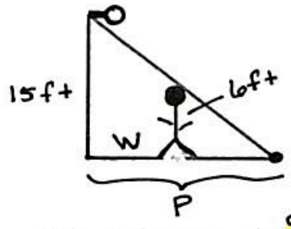
$\frac{dx}{dt} = 10 \text{ ft/sec}$

$\left.\frac{dz}{dt}\right|_{z=200} = ?$

$\left.\frac{dz}{dt}\right|_{z=200} = 5\sqrt{3} \text{ ft/sec}$

3. A street light is mounted at the top of a 15 foot pole. A man 6 feet tall walks away from the pole at a rate of 5 ft. per second.

(a) How fast is the tip of his shadow moving when he is 40 ft. from the pole?



$\frac{dW}{dt} = 5 \text{ ft/sec}$

find $\left.\frac{dP}{dt}\right|_{W=40}$

$\frac{15}{P} = \frac{6}{P-W}$

$6P = 15P - 15W$

$9P = 15W$

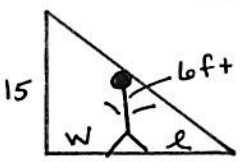
$9\frac{dP}{dt} = 15\frac{dW}{dt}$

$9\frac{dP}{dt} = 15(5)$

$\frac{dP}{dt} = \frac{25}{3} \text{ ft/sec}$

The tip of the shadow is increasing at a rate of 25/3 ft/sec.

(b) What is the rate of change of the length of his shadow when he is 40 ft. from the pole?



$\frac{dW}{dt} = 5 \text{ ft/sec}$

$\frac{15}{W+l} = \frac{6}{l}$

$15l = 6W + 6l$

$9l = 6W$

$9\frac{dl}{dt} = 6\frac{dW}{dt}$

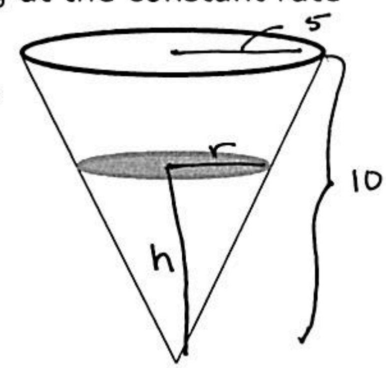
$9\frac{dl}{dt} = 6(5)$

$\frac{dl}{dt} = \frac{10}{3} \text{ ft/sec}$

The length of the shadow is increasing at a rate of $10/3$ ft/s.

4. A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.

$\frac{dh}{dt} = \frac{-3}{10} \frac{\text{cm}}{\text{hr}}$



{Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume of the container when h=5 cm.

Indicate units of measure.

$\frac{r}{h} = \frac{5}{10}$

$r = \frac{1}{2}h$

$V(5) = \frac{1}{3}\pi(2.5)^2 \cdot 5$

$= \frac{1}{3}\pi \cdot \frac{25}{4} \cdot 5 = \frac{125\pi}{12} \text{ cm}^3/\text{hr}.$

(b) Find the rate of change of the volume of the water in the container, with respect to time, when h = 5 cm. Indicate units of measure.

$V(t) = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(\overset{5}{\cancel{25}})\left(-\frac{3}{\underset{2}{\cancel{10}}}\right) = -\frac{15\pi}{8} \text{ cm}^3/\text{hr}$

When h=5, the volume is decreasing at a rate of $\frac{15\pi}{8}$ cm³/hr.