AP Calculus Name Key

3.7 Related Rates Day 1 Notes

Questions that ask for the calculation of the rate at which one variable changes, based on the rate at which another variable is known to change, are called related rates.

A "rate of change" is referring to an instantaneous rate of change, which is a derivative.

Practice translating sentences into proper calculus notation.

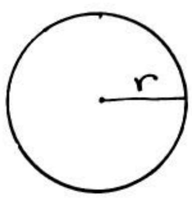
1) The area of a circle is increasing at a rate of six square inches per minute. $\frac{dA}{dt} = 6 \text{ in}^2/\text{min}$	3) The height of a tree is increasing at a rate of $\frac{1}{2}$ foot per year. $\frac{dh}{dt} = \frac{1}{2} \text{ ft/yr.}$
2) The volume of a cone is decreasing at a rate of two cubic feet per second. $\frac{dV}{dt} = -2 \text{ ft}^3/\text{sec}$	4) The water level in my fish tank is decreasing at a rate of 2 inches per hour. $\frac{d\ell}{dt} = -2 \text{ in/hr}$

Related rate problems require you to take a derivative with respect to time.
Differentiate each equation with respect to time.

1) $A = \pi r^2$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$	2) $V = \pi r^2 h$ * product rule $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$
3) $a^2 + b^2 = c^2$ $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$	4) $C = 2\pi r$ $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$
5) $V = s^3$ $\frac{dV}{dt} = 3s^2\frac{ds}{dt}$	6) $A = \frac{1}{2}bh$ * product rule $\frac{dA}{dt} = \frac{1}{2}b \cdot \frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt}$

Steps for Related Rates
1) Draw a picture.
2) Identify the variable whose rate of change you seek.
3) Find a formula relating the variables whose rate of change you seek with one or more variables whose rate of change you know.
4) Differentiate implicitly with respect to time t.
5) Substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.
6) Make sure that you have answered the question asked.

1) A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?



$\frac{dr}{dt} = 1$ ft/second (constant)

$A = \pi r^2$ ← formula

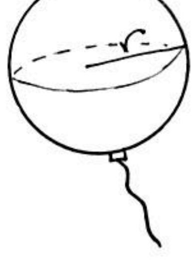
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ ← derive

Find $\left.\frac{dA}{dt}\right|_{r=4}$

$\left.\frac{dA}{dt}\right|_{r=4} = 2\pi(4)(1)$ ← plug in

When r=4, the area is increasing at a rate of 8π ft^2/sec.

3) Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.



$\frac{dV}{dt} = 4.5$ ft^3/min (constant)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Find $\left.\frac{dr}{dt}\right|_{r=2}$

$4.5 = 4\pi \cdot 4 \left.\frac{dr}{dt}\right|_{r=2}$

When the radius is 2 ft, the radius is changing at a rate of $\frac{9}{32\pi}$ ft/min.