

Notes – Differential Equations  
Section 6.3

name \_\_\_\_\_

1)  $\frac{dy}{dx} = 5x^4$        $(0, 2)$

2)  $\frac{dy}{dx} = \frac{2x}{y}$        $y(1) = -3$

3)  $y' = 9x^2y$     $y(0) = 2$

4)  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$        $y(0) = 0.5$

**Verifying Solutions to Differential Equations**

5) Determine whether the function is a solution of the differential equation  $y'' - y = 0$ .

a)  $y = \sin(x)$

b)  $y = 4e^{-x}$

6) Let  $f$  be a function with  $f(1) = 4$  such that for all points  $(x, y)$  on the graph of  $f$  the slope is given by  $\frac{3x^2 + 1}{2y}$ .

a) Find the slope of the graph of  $f$  at the point where  $x = 1$ .

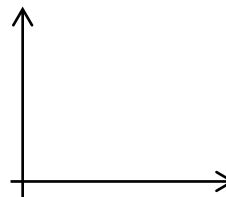
b) Write an equation for the line tangent to the graph of  $f$  at  $x = 1$  and use it to approximate  $f(1.2)$ .

c) Find  $f(x)$  by solving the separable differential equation  $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$  with the initial condition  $f(1) = 4$ .

## 7) Exponential Growth and Decay

The rate of growth is directly proportional to the population.

$$\frac{dP}{dt} = kP$$



Example: A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If **the weight** of the puppy during its first 6 months **is increasing at a rate proportional to its weight**, then how much will the puppy weigh when it is 3 months old?

**Worksheet – Differential Equations**  
**Section 6.3**

name \_\_\_\_\_

Find the particular equation for each differential equation and initial condition.

1)  $\frac{dy}{dx} = y^2$        $y(1) = -2$

2)  $\frac{dy}{dx} = \frac{x^3}{y^2}$        $y(1) = 2$

3)  $\frac{dy}{dx} = \frac{1}{x}$        $(1, 2)$

4)  $\frac{dy}{dx} = \frac{1}{3y^2}$        $(2, -2)$

5)  $\frac{dy}{dx} = \frac{2x}{y}$        $(2, 4)$

6)  $\frac{dy}{dx} = \frac{1+x}{xy}$        $y(1) = -4$

7)  $\frac{dy}{dx} = \frac{e^x}{y}$        $y(0) = 4$

8)  $xy \frac{dy}{dx} - \ln x = 0$        $y(1) = 0$

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Verify whether or not the following are solutions to the given differential equations:

9) Differential

$$y'' + y = 0$$

$$y'' + 4y' = 2e^x$$

Potential Solution

$$y = C_1 \sin x - C_2 \cos x$$

$$y = \frac{2}{5}(e^{-4x} + e^x)$$

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10) Given  $\frac{dy}{dx} = \frac{-4x+2}{y}$ .

a) Write the equation for the line tangent to the graph at  $(2, -4)$  and use it to approximate  $f(1.8)$ .

b) Find the particular solution  $y = f(x)$  to the differential equation with initial condition  $f(2) = -4$ .

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11) Population  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. Find the population in 2013 if the population was 12,000 in 1980 and the population doubles every 10 years.

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12) The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1599 years. What percent of a present amount will remain after 25 years?