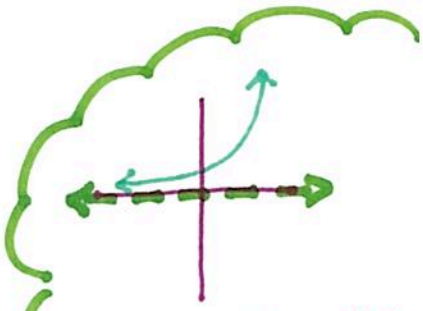
More Chain Rule!

(Exponential & Log Functions)



$a^x \cdot a^y = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{x \cdot y}$

Exponentials:

$f(x) = a^x \longrightarrow f'(x) = a^x \cdot \ln a$

$f(x) = a^u \longrightarrow f'(x) = a^u \cdot du \cdot \ln a$

$f(x) = e^x \longrightarrow f'(x) = e^x$

$f(x) = e^u \longrightarrow f'(x) = e^u \cdot du$

examples:

1) $y = e^{3x+2}$

$u = 3x+2$

$du = 3$

$y' = e^{3x+2} \cdot 3 = 3e^{3x+2}$

2) $y = 2e^{\sin x}$

$u = \sin x$

$du = \cos x$

$y' = 2e^{\sin x} \cdot \cos x$

$= 2\cos x\, e^{\sin x}$

3) $y = 6^{\sec x}$

$u = \sec x$

$du = \sec x \tan x$

$$y' = 6^{\sec x} \cdot \sec x \tan x \cdot \ln 6$$

4) $y = 2x \cdot 3^{5x}$

Product Rule!

$$y' = 2x \cdot (3^{5x} \cdot 5 \cdot \ln 3) + 3^{5x} \cdot 2$$

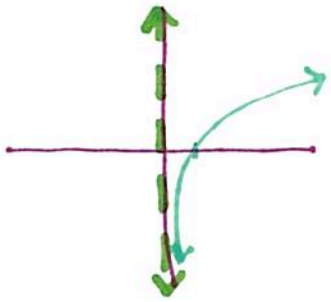
$$y' = 2 \cdot 3^{5x} (5x \ln 3 + 1)$$

5) $y = \dfrac{e^x}{e^x + 1}$ Quotient Rule!

$$y' = \frac{(e^x+1) \cdot e^x - e^x \cdot e^x}{(e^x+1)^2} = \frac{e^{2x} + e^x - e^{2x}}{(e^x+1)^2}$$

$$y' = \frac{e^x}{(e^x+1)^2}$$

Logs!



$y = \log_a x$

Properties:

$\ln(x \cdot y) = \ln x + \ln y$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\ln x^y = y \cdot \ln x$

$\ln 1 = 0$

$\ln e = 1$

Derivatives:

$f(x) = \log_a x \longrightarrow f'(x) = \frac{1}{x \ln a}$

$f(x) = \log_a u \longrightarrow f'(x) = \frac{du}{u \ln a}$

$f(x) = \ln x \longrightarrow f'(x) = \frac{1}{x}$

$f(x) = \ln u \longrightarrow f'(x) = \frac{du}{u}$

examples:

1) $y = \ln(x^2+1)$

$u = x^2+1$

$du = 2x$

$$y' = \frac{2x}{x^2+1}$$

2) $y = \ln [x \cdot \sin x]$

$y = \ln x + \ln \sin x$ $\quad u = \sin x$, $du = \cos x$

$y' = \frac{1}{x} + \frac{\cos x}{\sin x}$

$y' = \frac{1}{x} + \cot x$

3) $y = \log_3 \left(\frac{x+1}{2x-3}\right)$

$y = \log_3 (x+1) - \log_3 (2x-3)$

$y' = \frac{1}{(x+1)\ln 3} - \frac{2}{(2x-3)\ln 3}$

4) $y = \ln \sqrt{\frac{x}{\tan x}}$

$y = \frac{1}{2}[\ln x - \ln \tan x]$

$y' = \frac{1}{2}\left(\frac{1}{x} - \frac{\sec^2 x}{\tan x}\right)$

5) $y = e^{\ln (5x)} \rightarrow e^{\ln u} = u$

$y = 5x$

$y' = 5$