AP Calculus Name – Key 2019 –

Motion – Using an Equation

no units

Y=

A particle moves up and down the y-axis with velocity given by the equation $v(t) = \frac{1}{3}t^3 - 3t^2 + 8t - \frac{16}{3}$ during the time interval $0 \le t \le 5$. At time t = 0, its position is y = 1.

1) Where is the particle at $t = \frac{3}{2}$? $1 + \int_0^{3/2} v(t)\,dt = .953$

Sentence: At $t = 3/2$, the particle is at .953.

2) Find the acceleration of the particle at t = 3. $a(t) = v'(t)$

$v'(3) = -1$ $v' = t^2 - 6t + 8$

Sent: At $t = 3$, the velocity is decreasing by 1.

3) At what time is the particle at rest? At $t = 3$, the acceleration is -1.

$v(t) = 0$ at $t = 1, 4$ on table

At $t = 1$ and $t = 4$, the particle is at rest.

4) Is the particle moving up or down the y-axis at time $t = \frac{1}{2}$? What is the speed of the particle at that time?

$v(\frac{1}{2}) = -2.041$ – moving down

Speed = 2.041

At $t = 1/2$, the particle is moving down at a speed of 2.041.

5) What is the average velocity from t = 0 to t = 5?

Avg. Velocity $= \frac{1}{5}\int_0^5 v(t)\,dt = .083$

From $t = 0$ to $t = 5$, the average velocity is .083.

6) What is the maximum velocity of the particle? When does this happen?

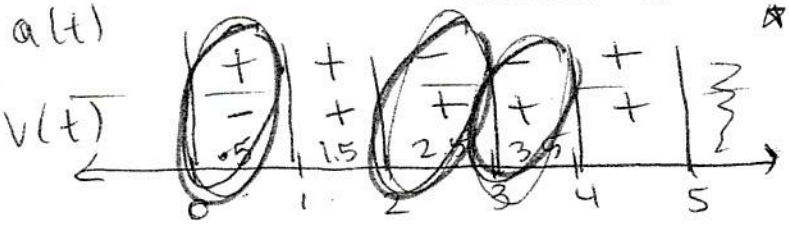
candidates test
crit. values & endpts
CV: $v' = 0$ or dne

t	v(t)
0	−16/3
2	4/3
4	0
5	4/3

$t^2 - 6t + 8$
$(t-2)(t-4)$
$t = 2, 4$

At $t = 2$ and $t = 5$, the particle has a max velocity of 4/3.

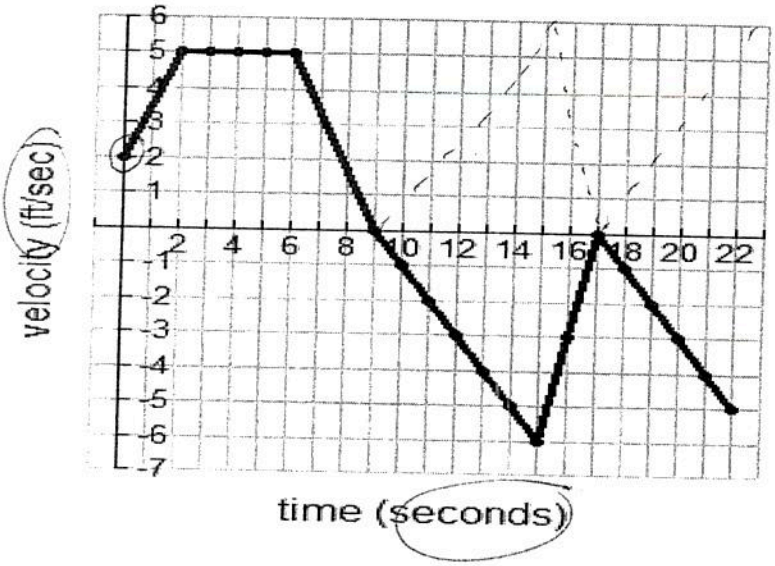
7) When is the speed of the particle decreasing $0 \le t \le 5$?



* On $0 \le t \le 5$, the speed of the particle is decreasing $(0,1) \cup (2,4)$.

AP Calculus

Motion – Using a Graph



position: ft
velocity = ft/sec
accel. = ft/sec^2

A particle moves along the x-axis with velocity as shown in the graph above. Its position, x(t), at t = 0 is 5.

1) At t = 0, is the particle moving left or right?

$v(0) = 2$

JUSTIFY

At t=0 seconds, the particle is moving right. since $v(t) > 0$.

2) When is the particle at rest? $v(t) = 0$

At t=9 seconds and 17 seconds, the particle is at rest. since $v(t) = 0$.

3) When does the particle change direction?

At t=9 seconds, the particle changes directions.

4) When is the particle's speed the greatest? speed $= |v(t)|$ ∴ 15 seconds

At t=15 seconds, the particle's speed in ft. per second, is the greatest.

5) What is the acceleration of the particle at t = 1?

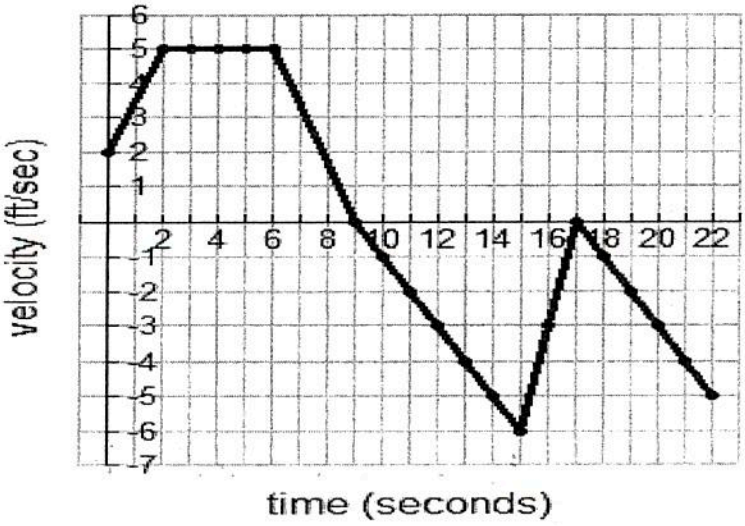
$$a(t) = v'(t) = \frac{v(2) - v(0)}{2 - 0} = \frac{3}{2} \text{ ft./sec}^2$$

At t=1 second, the accel. of the particle is 3/2 feet per second.

6) When is acceleration zero?

$a(t)=0$ when $v'(t)=0$

on $(2,6)$ the acceleration is zero since $v'(t)=0$.



* calc

7) What is the total distance traveled from t = 0 to t = 15?

total dist $= \int_0^{15} |v(t)|\,dt$

algebra.

$$\frac{2+5}{2}(2) + 4(5) + \frac{(3)(5)}{2} + \frac{(6)(6)}{2} = 52.5 \text{ ft.}$$

From $t=0$ to $t=15$, the total distance travelled is 52.5 feet

8) What is the particle's position at t = 15?

(Previous)

$x(t)$ at $t=0$ is 5

$$5 + \int_0^{15} v(t)\,dt = 5 + 7 + 20 + 7.5 - 18 = \boxed{21.5 \text{ ft.}}$$

Beginning Displacement

9) When is the particle farthest to the right? → Abs Max position ↙ pos.

At $t=9$, the particle is farthest to the right.

t	$x(t)$
0	5
9	39.5
22	3

$39.5 = 5 + \int_0^9 v(t)$ → only are changing from + to −

$= 39.5 - 24$

$= 15.5 - 12.5$

10) When is the speed of the particle increasing from $0 \le t \le 22$?

When $v(t)$ and $a(t)$ have same signs

$(0,2) \cup (9,15) \cup (17,22)$ seconds

On $0 \le t \le 22$, the intervals $(0,2) \cup (9,15) \cup (17,22)$ seconds are increasing their speed in ft./second^2

AP Calculus

Where is speed the greatest? $|v(t)|$ @ $t = 40$ What would 51 Bc? (Abs Max) accel = 0

Motion – Using a Table

The velocity of a car is recorded at 10 second intervals. We can assume that the function and its derivative are continuous and differentiable over the entire interval. Position at t = 0 is 0 feet.

Time (seconds)	0	10	20	30	40	50	60
Velocity (ft/sec)	0	38	42	48	51	50	45

1) Approximately, how far did the car travel during the last 30 seconds using a left sum and 3 subintevals?

$(48 \cdot 10) + (51 \cdot 10) + (50 \cdot 10) = 1490$ ft.

college board ok

2) From the data in the chart and assuming that all critical numbers are represented, during what time interval is acceleration positive?

$a(t) > 0$ when $v'(t)$ is ... (t) is inc.

$(0, 40)$ sec. $v(t)$ is inc.

On the interval (0,40) seconds, acceleration is positive.

3) Approximate the average velocity from t = 10 to t = 40 using a right sum and n = 3.

$$\frac{1}{40-10}\int_{10}^{40} v(t)\,dt \approx \frac{51 \cdot 10 + 48 \cdot 10 + 42 \cdot 10}{30} \approx 47 \text{ ft./sec}$$

From $t = 10$ to $t = 40$ seconds, the approximate avg. velocity is 47 ft per second.

4) Approximate the acceleration at time t = 50.

$$a(50) \approx \frac{v(60) - v(40)}{60 - 40} \approx \frac{45 - 51}{20} \approx \frac{-3}{10} \text{ ft./sec.}^2$$

At $t = 50$, the acceleration is $-\frac{3}{10}$ feet per second per second.

5) Using a midpoint sum and 3 equal partitions, approximate where the car is at the end of 60 seconds.

$$P(60) \approx P(0) + \int_0^{60} v(t)\,dt \approx 0 + (38 \cdot 20) + (48 \cdot 20) + (50 \cdot 20)$$

≈ 2120 ft.

At the end of 60 seconds, the car is at approx. 2720 ft.

velocity

6) Using correct units, explain the meaning of $\frac{1}{60}\int_0^{60} v(t)\,dt$.

$\frac{1}{60}\int_0^{60} v(t)\,dt$ is from $t = 0$ to $t = 60$ seconds, the average ~~of the car~~ in ft. per second. ~~duration~~

You must round!