Warm Up on Inverses:

1. Find the inverse of f(x) = 2x - 4.

$y = 2x - 4$

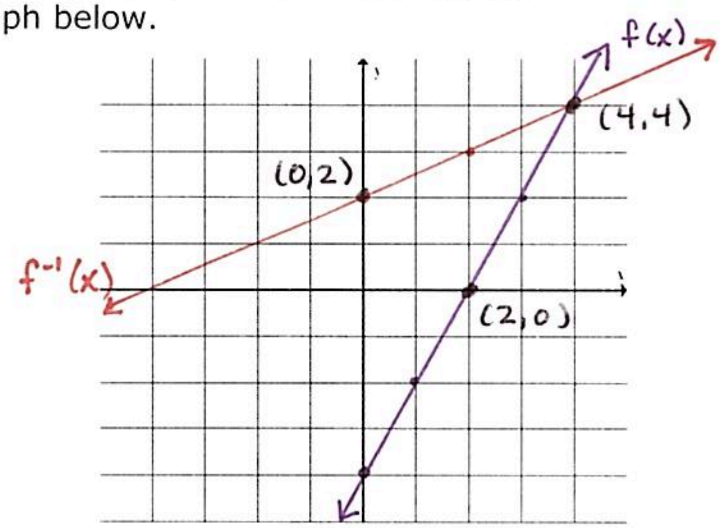
$x = 2y - 4$

$x + 4 = 2y$

$y = \frac{x+4}{2}$

$f^{-1} = \frac{1}{2}x + 2$

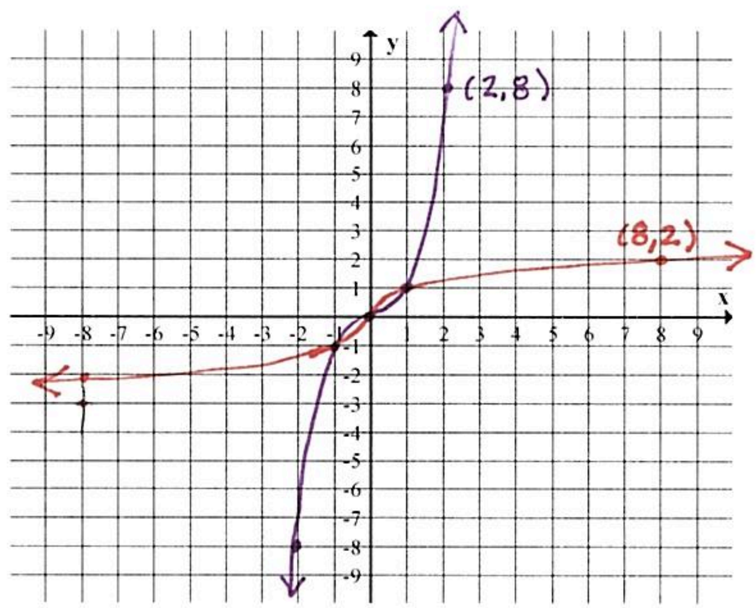
2. Graph f(x) and its inverse on the graph below.



3. Find f(2) = 0

4. Find $f^{-1}(0)$ = 2

The slopes of inverse functions –



1. On the given coordinate plain, graph $f(x) = x^3$.

   Mark the point (2,8) clearly.

   What is the inverse of your function?

   $f^{-1} = \sqrt[3]{x}$

   Find f ′(2). $f'(x) = 3x^2$

   $f'(2) = 12$

2. On the given coordinate plain, graph $g(x) = \sqrt[3]{x}$. $x^{1/3}$

   Mark the point (8,2) clearly.

   Find g′(8).

   $g'(x) = \frac{1}{3}x^{-2/3}$

   $g'(x) = \frac{1}{3\sqrt[3]{x^2}}$

   $g'(8) = \frac{1}{12}$

What do you notice about f′(2) and g′(8)? Reciprocals!

If f(x) and g(x) are inverses of each other, and f(a)=b, then $g'(b) = \frac{1}{f'(a)}$

Steps.

1. Find the inverse point if it is not already given.
2. Find f'(x)
3. Find f'(a)
4. $g'(b) = \frac{1}{f'(a)}$

Example: $f(x)=x^2+2x+1$; $x>0$

g(x) is the inverse of f(x).

Find g'(4).

① $4 = x^2 + 2x + 1$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = \cancel{-3}, 1$

$x>0$

$f(1) = 4$

② $f'(x) = 2x + 2$

③ $f'(1) = 2 + 2 = 4$

④ $g'(4) = \frac{1}{4}$

Example: $f(x)=x^3+3x^2+5x$

g(x) is the inverse of f(x).

Find g'(-3).

① $-3 = x^3 + 3x^2 + 5x$

$0 = x^3 + 3x^2 + 5x + 3$

NOT FACTORABLE!

Possibles: $\pm 1, 3$

-1	1	3	5	3
	↓	-1	-2	-3
	1	2	3	0 :)

$(x+1)(\cancel{x^2+2x+3}) = 0$

imaginary

$x = -1$

$f(-1) = -3$

② $f'(x) = 3x^2 + 6x + 5$

③ $f'(-1) = 3 - 6 + 5 = 2$

④ $g'(-3) = \frac{1}{2}$

Example: $f(x)=x^3-2x+1$

g(x) is the inverse of f(x).

Find g'(1) if g(1)=2.

① $f(2) = 1$

② $f'(x) = 3x^2 - 2$

③ $f'(2) = 12 - 2 = 10$

④ $g'(1) = \frac{1}{10}$

Optional Method: (use implicit)

$f(x) = x^2 + 2x + 1$ find $g'(4)$ if $f(1) = 4$

$x = y^2 + 2y + 1$

$1\left(\frac{dx}{dx}\right) = 2y\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right)$

$1 = \frac{dy}{dx}(2y + 2)$

$\frac{dy}{dx} = \frac{1}{2y+2}$

$\left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{2+2} = \frac{1}{4}$

Monotonic Functions:

A function is monotonic if it is always increasing or always decreasing.

$f'(x) \geq 0$ or $f'(x) \leq 0$ for all x.

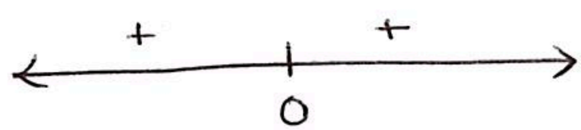
example.

Is $f(x) = x^3+1$ monotonic? Justify

$f'(x) = 3x^2$

$0 = 3x^2$

$x = 0$



$f'(x) \geq 0$ for all x. $\therefore$ $f(x)$ is monotonic.