AP Calculus AB

Name Key

Worksheet (Derivatives of Inverse Functions)

1. For which of the following increasing functions f does $\left(f^{-1}\right)'(20) = \frac{1}{5}$?

(A) $f(x) = x + 5$ $f' \neq 5$

$f' = 1$

$(f^{-1})'(20) = \frac{1}{5}$

$f'(0) = 5$

(B) $f(x) = x^3 + 5x + 20$

$f' = 3x^2 + 5$

$f(0) = 20$

(C) $f(x) = x^5 + 5x + 14$ $f(0) \neq 20$

$f' = 5x^4 + 5$

(D) $f(x) = e^x + 5x + 19$ $f' \neq 5$

$f' = e^x + 5$

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2. Let f and g be inverse functions that are differentiable for all x. If $f(-5) = 7$ and $g'(7) = 3$, which of the following statements must be false?

I. $f'(3) = -\frac{1}{3}$ False

II. $f'(-5) = \frac{1}{3}$ True

III. $f'(7) = \frac{1}{3}$ Maybe

$f + g$ must be 1 to 1

since $g'(7) > 0$

$f' + g'$ must always be > 0

(A) I only

(B) II only

(C) III only

(D) I and III only

3) Let f be the function defined by $f(x)=\frac{1}{27}\left(x^5+2x^3\right)$. If $g(x)=f^{-1}(x)$ and $g(-11)=-3$, what is the value of $g'(-11)$?

$f(-3)=-11$

$g'(-11)=\frac{1}{f'(-3)}=\frac{1}{17}$

$f'=\frac{1}{27}\left(5x^4+6x^2\right)$

$f'(-3)=\frac{1}{27}\left(5\cdot 81+6\cdot 9\right)$

$=15+2=17$

4) Let f be the function defined by $f(x)=x^3-\frac{4}{x}$. If $g(x)=f^{-1}(x)$ and $g(6)=2$, what is the value of $g'(6)$?

$f(2)=6$

$g'(6)=\frac{1}{f'(2)}=\frac{1}{13}$

$f'=3x^2+\frac{4}{x^2}$

$f'(2)=12+1=13$

5) Let f be the function defined by $f(x)=\sqrt{x-4}$. If $g(x)=f^{-1}(x)$ and $g(2)=8$, what is the value of $g'(2)$?

$f(8)=2$

$g'(2)=\frac{1}{f'(8)}=4$

$f'=\frac{1}{2}(x-4)^{-1/2}=\frac{1}{2\sqrt{x-4}}$

$f'(8)=\frac{1}{2\sqrt{4}}=\frac{1}{4}$

6) Let f be the function defined by $f(x)=\frac{4}{1+x^2}$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

$f=4(1+x^2)^{-1}$

$f(1)=2$

$f'=-4(1+x^2)^{-2}\cdot 2x$

$=\frac{-8x}{(1+x^2)^2}$

$f'(1)=\frac{-8}{4}=-2$

$\therefore g'(2)=-\frac{1}{2}$

7) Let f be the function defined by $f(x)=3-4x$. If $g(x)=f^{-1}(x)$ and $g(-1)=1$, what is the value of $g'(-1)$?

$f'=-4$ (a constant)

$\therefore g'(-1)=-\frac{1}{4}$