Reimann Sums HWK name ________________________

1) Which of the following is the midpoint Riemann sum approximation of $\int_4^6 \sqrt{x^3+1}\,dx$ using 4 subintervals of equal width?

$\Delta x = \frac{6-4}{4} = \frac{1}{2}$

~~A~~ $\frac{1}{4}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

B $\frac{1}{2}\left(\sqrt{4.25^3+1}+\sqrt{4.75^3+1}+\sqrt{5.25^3+1}+\sqrt{5.75^3+1}\right)$

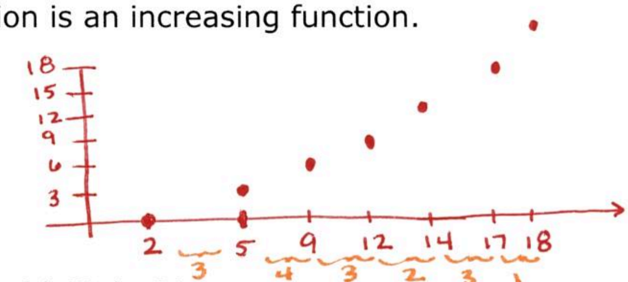
~~C~~ $\frac{1}{4}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$

D $\frac{1}{2}\left(\frac{\sqrt{4^3+1}+\sqrt{4.5^3+1}}{2}+\frac{\sqrt{4.5^3+1}+\sqrt{5^3+1}}{2}+\frac{\sqrt{5^3+1}+\sqrt{5.5^3+1}}{2}+\frac{\sqrt{5.5^3+1}+\sqrt{6^3+1}}{2}\right)$ TRAPEZOID

2) Approximate the area of the region bounded by f(x) from x = 2 to x = 18 using 6 subintervals as indicated in the chart. Assume the function is an increasing function.

3 4 3 2 3 1

x	2	5	9	12	14	17	18
f(x)	0	2	6	9	13	18	21



a) Left Riemann sum:

$0\cdot 3 + 2\cdot 4 + 6\cdot 3 + 9\cdot 2 + 13\cdot 3 + 18\cdot 1$

b) Right Riemann sum:

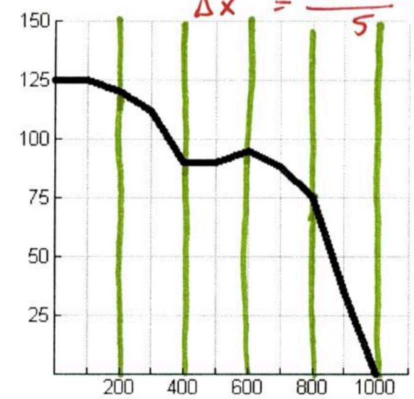
$2\cdot 3 + 6\cdot 4 + 9\cdot 3 + 13\cdot 2 + 18\cdot 3 + 21\cdot 1$

c) Trapezoid sum:

$\frac{0+2}{2}\cdot 3 + \frac{2+6}{2}\cdot 4 + \frac{6+9}{2}\cdot 3 + \frac{9+13}{2}\cdot 2 + \frac{13+18}{2}\cdot 3 + \frac{18+21}{2}\cdot 1$

3) Use the graph and chart to the right.

$\Delta x = \frac{1000-0}{5} = 200$



a) Estimate the area using right Riemann sums with 5 equal width rectangles.

$200 \cdot 125 + 200 \cdot 120 + 200 \cdot 90 + 200 \cdot 95 + 200 \cdot 75$

SORRY

b) Estimate the area using left Riemann sums with 5 equal width rectangles.

$200 \cdot 120 + 200 \cdot 90 + 200 \cdot 95 + 200 \cdot 75 + 200 \cdot 0$

	x	y	
L	0	125	
M	100	125	
L	200	120	R
M	300	112	
L	400	90	R
M	500	90	
L	600	95	R
M	700	88	
L	800	75	R
M	900	35	
	1000	0	R

c) Estimate the area using midpoint Riemann sums with 5 subintervals of equal length.

$200 \cdot 125 + 200 \cdot 112 + 200 \cdot 90 + 200 \cdot 88 + 200 \cdot 35$

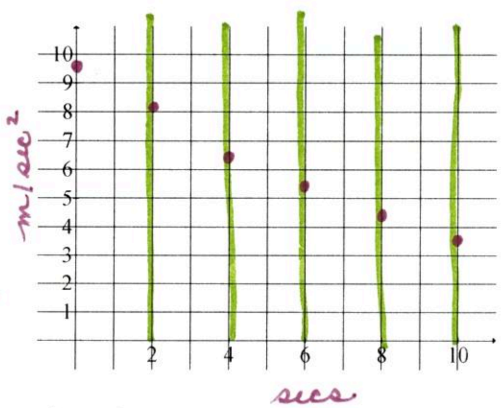
d) Estimate the area using the Trapezoid Rule with 5 intervals of equal length.

$\frac{125+120}{2} \cdot 200 + \frac{120+90}{2} \cdot 200 + \frac{90+95}{2} \cdot 200 + \frac{95+75}{2} \cdot 200 + \frac{75+0}{2} \cdot 200$

4) You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table below gives your acceleration in m/sec^2 after t seconds.

Time (sec)	0	2	4	6	8	10
Acceleration (m/sec^2)	9.81	8.03	6.53	5.38	4.41	3.61

1.78 1.5 1.15 .97 .8



a. Give upper and lower estimates of your speed at t=10 (use Δt=2).

upper (left) = $9.81(2) + 8.03(2) + 6.53(2) + 5.38(2) + 4.41(2)$ m/s

lower (right) = $8.03(2) + 6.53(2) + 5.38(2) + 4.41(2) + 3.61(2)$ m/s

b. Use the trapezoid method to estimate your speed at t=10. What does the concavity of the graph of acceleration tell you about your estimate?

TRAP: $\frac{9.81+8.03}{2} \cdot 2 + \frac{8.03+6.53}{2} \cdot 2 + \frac{6.53+5.38}{2} \cdot 2 + \frac{5.38-4.41}{2} \cdot 2 + \frac{4.41+3.61}{2} \cdot 2$ m/s

a(t) is concave up (decreasing at a decreasing rate), ∴ this is an over estimate of the actual velocity.

5) In order to determine the average temperature for the day, a meteorologist decides to record the temperature at eight times during the day. She further decides that these recordings do not have to be equally spaced during the day because she does not need to make several readings during those periods when the temperature is not changing much (as well as not wanting to get up in the middle of the night.) She decides to make one reading at some time during each of the intervals in the table below.

Time	12AM-5AM	5AM-7AM	7AM-9AM	9AM-1PM	1PM-4PM	4PM-7PM	7PM-9PM	9PM-12AM
Temp	42°	57°	72°	84°	89°	75°	66°	52°

a. Using a Riemann sum, calculate the average temperature for the day.

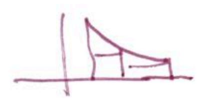
$$\frac{5\cdot 42 + 2\cdot 57 + 2\cdot 72 + 4\cdot 84 + 3\cdot 89 + 3\cdot 75 + 2\cdot 66 + 3\cdot 52}{24} \ {}^\circ F$$

6)

a^2 $3a-a^2$ $3a$ a

x	0	a^2	$3a$	$6a$	$7a$
$f(x)$	1	-1	-3	-7	-9

$f(x)$ is decreasing



The continuous function f is decreasing for all x. Selected values of f are given in the table above, where a is a constant with $0 < a < 3$. Let R be the right Riemann sum approximation for $\int_0^{7a} f(x)\,dx$ using the four subintervals indicated by the data in the table. Which of the following statements is true?

A $R = (a^2 - 0)\cdot 1 + (3a - a^2)\cdot(-1) + (6a - 3a)\cdot(-3) + (7a - 6a)\cdot(-7)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$

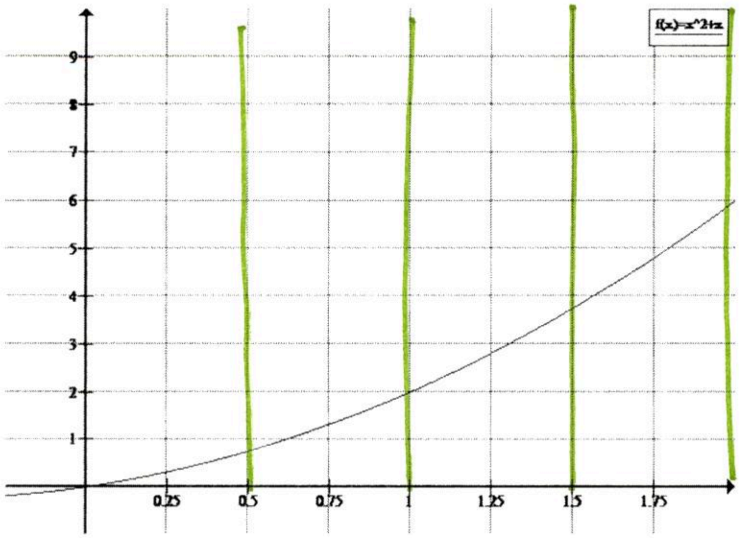
B $R = (a^2 - 0)\cdot 1 + (3a - a^2)\cdot(-1) + (6a - 3a)\cdot(-3) + (7a - 6a)\cdot(-7)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$

C $R = (a^2 - 0)\cdot(-1) + (3a - a^2)\cdot(-3) + (6a - 3a)\cdot(-7) + (7a - 6a)\cdot(-9)$ and is an underestimate for $\int_0^{7a} f(x)\,dx$

D $R = (a^2 - 0)\cdot(-1) + (3a - a^2)\cdot(-3) + (6a - 3a)\cdot(-7) + (7a - 6a)\cdot(-9)$ and is an overestimate for $\int_0^{7a} f(x)\,dx$

7) Let $f(x) = x^2 + x$. Consider the region bounded by the graph of f, the x-axis and the line x=2. Divide the interval [0,2] into 4 equal subintervals.

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$



	x	f(x)	
L	0	0	
	.25	.3125	M
L	.5	.75	R
	.75	1.3125	M
L	1	2	R
	1.25	2.8125	M
L	1.5	3.75	R
	1.75	4.8125	M
	2	6	R

a. Obtain a lower estimate for the area of the region by using left endpoints.

lower (left) = $\frac{1}{2}(0) + \frac{1}{2}(.75) + \frac{1}{2}(2) + \frac{1}{2}(3.75)$

b. Obtain an upper estimate by using right endpoints.

upper (right) = $\frac{1}{2}(.75) + \frac{1}{2}(2) + \frac{1}{2}(3.75) + \frac{1}{2}(6)$

c. Find an approximation for the area using trapezoids. Is your estimate too big or too small. Why?

$\frac{0+.75}{2} \cdot \frac{1}{2} + \frac{.75+2}{2} \cdot \frac{1}{2} + \frac{2+3.75}{2} \cdot \frac{1}{2} + \frac{3.75+6}{2} \cdot \frac{1}{2}$

$f' = 2x+1$

$f'' = 2 > 0 \therefore$

f is cc up

since f is cc↑, the trapezoidal sum is an over estimate

d. Obtain an estimate for the area using midpoints.

$\frac{1}{2}(.3125) + \frac{1}{2}(1.3125) + \frac{1}{2}(2.8125) + \frac{1}{2}(4.8125)$