Calculus AB
Sections ~~3.1-3.6~~ Curve Sketching & Theorems Name Key
Review for Test

1. Given that $f(x)=x^3-3x^2+12$ on the interval [-2,4]

a. Find all critical numbers of f.

$f' = 3x^2 - 6x$

$0 = 3x(x-2)$

CN @ $x = 0, 2$

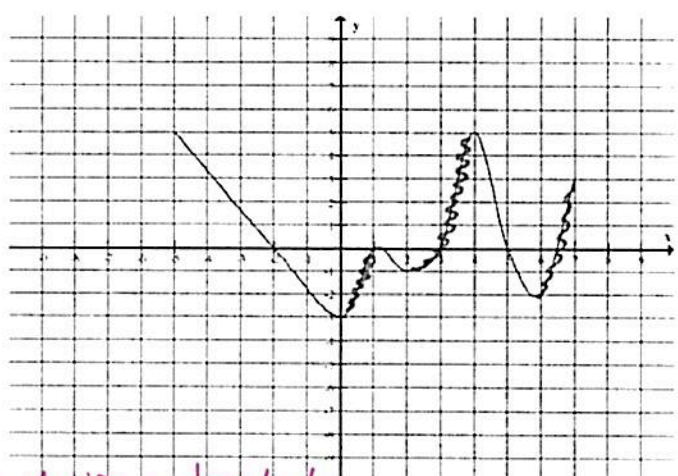
b. Find the absolute extrema of f.

x	f
-2	-8
0	12
2	8
4	28

Absolute Min = -8 at $x = -2$

Absolute Max = 28 at $x = 4$

2. The graph of f′, the derivative of f is sketched below. Use this graph to answer the following questions.



a. Over what intervals is f increasing and decreasing.

increasing $(-5,-2)\cup(3,5)\cup(6.5,7)$

decreasing $(-2,3)\cup(5,6.5)$

b. Over what intervals is f concave up and concave down.

Concave up $(0,1)\cup(2,4)\cup(6,7)$

Concave down $(-5,0)\cup(1,2)\cup(4,6)$

c. Find all extrema and points of inflection of f.

Rel min @ $x = 3, 6.5$ → means relative AND absolute

Rel max @ $x = -2, 5$

POI @ $x = 0, 1, 2, 4, 6$

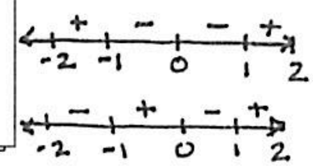
We do not have the f(x) equation so we can't find y-values of extrema. Giving x-values is fine

Abs Max at $x = -2$, $x = 5$, $x = 7$ (we can't know which)

Abs Min at $x = 3$, $x = 6.5$, $x = -5$ (we can't know which one)

3. Let f be a function that is continuous on the interval [-2,2]. The function f is twice differentiable. The function f and its derivatives have the properties indicated in the table below. Given the information in the chart below answer the following questions:

See chart on original packet.



a. Find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

Rel min @ x = 1 → f' changes from − to +

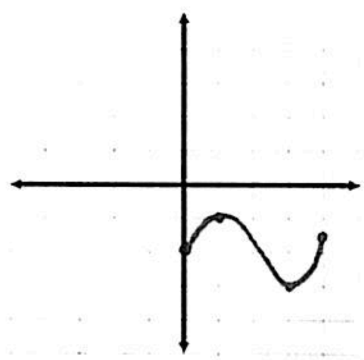
Rel. max @ x = −1 → f' changes from + to −

b. Find all values of x at which f has a point of inflection. Justify your answer.

POI @ x = −1, 0, 1 → f'' changes sign

4. Sketch the graph of a function f that is continuous on $[0,3]$ and has the given properties:

Absolute maximum at x = 1,
Absolute minimum at x = 2



5. Find the absolute maximum and absolute minimum values of f on the given interval.
$f(x) = 3x - \cos x$ $[-\pi, \pi]$ Show all the calculus that leads to your conclusion.

$f' = 3 + \sin x$

$0 = 3 + \sin x$

NO CRITICAL VALUES

x	f	
$-\pi$	$-3\pi+1$	← Min
π	$3\pi+1$	← Max

Abs max $(\pi, 3\pi+1)$
Abs min $(-\pi, -3\pi+1)$

6. Given the function $f(x) = x - 3x^{\frac{1}{3}}$ find the following: You must show all the calculus that leads to your conclusions.

A) The intervals of increase and decrease.

$f' = 1 - x^{-2/3}$

$f' = 1 - \frac{1}{x^{2/3}}$

$f' = \frac{x^{2/3} - 1}{x^{2/3}}$

CN @ $x = \pm 1, 0$

$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

increasing $(-\infty, -1) \cup (1, \infty)$

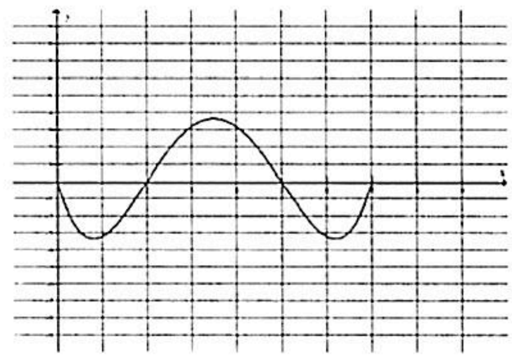
decreasing $(-1, 1)$

B) The local maximum and minimum values.

Rel max @ $x = -1$ is $-1 + 3 = 2$

Rel min @ $x = 1$ is $1 - 3 = -2$

7. The graph of the first derivative of a function f is shown



A) On what intervals is f increasing? Explain your answer.

$(2, 5)$ $f' > 0$

B) At what values of x does f have a local maximum? a local minimum? Explain your answers.

Rel max @ $x = 5 \rightarrow f'$ changes from + to −

Rel min @ $x = 2 \rightarrow f'$ changes from − to +

C) At what values of x does f have points of inflection? Justify your answer.

POI @ $x = 1, 3.5, 6 \rightarrow$ the slope changes sign

8. Given the function $f(x) = x^3 - 12x + 1$

a) Find the intervals on which f is concave up or concave down. Show all the calculus that leads to your conclusion.

$f' = 3x^2 - 12$

$f'' = 6x$

$0 = 6x$

PPOI @ $x = 0$

$- \quad +$

0

concave ↓ $(-\infty, 0)$

concave ↑ $(0, \infty)$

b) Find any points of inflection. Justify your answer.

POI @ $x = 0$

$(0, 1)$ f'' changes signs

9. Determine if the Mean Value Theorem applies to the function $f(x) = 2 - x^2$ on the interval $\left[0, \sqrt{2}\right]$. If so, find **ALL** points (x-values only) that are guaranteed to exist by the Mean Value Theorem.

$f'(x) = -2x$

a) No. Mean Value Theorem does not apply.

$f(0) = 2 - 0^2 = 2$

b) Yes: $x = -\frac{2}{\sqrt{2}} + 2$

$f(\sqrt{2}) = 2 - \sqrt{2}^2 = 2 - 2 = 0$

c) Yes: $x = \frac{1}{\sqrt{2}}$

$AROC = \frac{2-0}{0-\sqrt{2}} = \frac{2}{-\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{-2} = -\sqrt{2}$

d) Yes: $x = \pm\frac{1}{\sqrt{2}}$

When does iroc = aroc?

$\frac{-2x}{-2} = \frac{-\sqrt{2}}{-2}$

$x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

10. If f′(4) = 0 and f″(4) = 3, then there is a _______________ at x = 4.

a) relative maximum

b) relative minimum

c) point of inflection

critical #

concave up

11. f(x) is continuous over the interval [-3,7] and differentiable over the interval (-3,7). The average rate of change over the interval [-3,7] is 4. It is determined that the instantaneous rate of change for f(x) is 4 at x-values of -3, 2, and 8. What are values of *c* that satisfy the conclusion of the Mean Value Theorem on the interval [-3,7]?

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MVT promises c-values on the open interval. Even though the question asked for values [-3, 7], we can only get c values from (-3, 7).