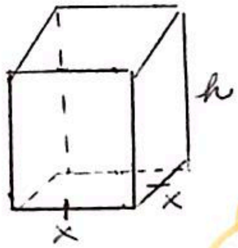
Calculus AB
Optimization HWK

Name: Key

An open storage bin with a square base and vertical sides is to be constructed from 108 square feet of aterial. Determine its dimensions if its volume is to be a maximum.



$SA = x^2 + 4xh$

$108 = x^2 + 4xh$

$h = \frac{108 - x^2}{4x}$

$V = x^2 \cdot h$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$V = \frac{108}{4}x - \frac{x^3}{4}$

$V' = 27 - \frac{3}{4}x^2$

$V' = 0$ when

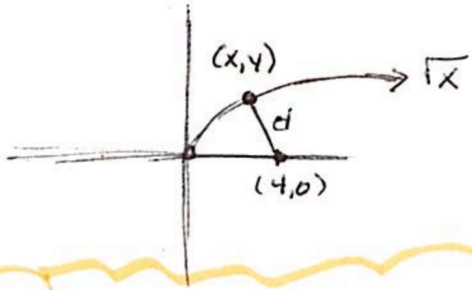
$x = 6$

$\xleftrightarrow{\quad + \quad 6 \quad - \quad}$

Rel max

The dimensions of the box would be 6 ft × 6 ft × 3 ft.

2. Find the point (x, y) on the graph of $y = \sqrt{x}$ $\overset{\text{min}}{\text{nearest}}$ the point (4, 0).



$d = \sqrt{(4-x)^2 + (0-y)^2}$

$d = \sqrt{(4-x)^2 + (0-\sqrt{x})^2}$

$d = \sqrt{16 - 8x + x^2 + x}$

$d = (x^2 - 7x + 16)^{1/2}$

$d' = \frac{1}{2}(x^2 - 7x + 16)^{-1/2}(2x - 7)$

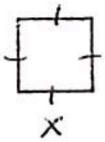
$d' = 0$ when $x = 3.5$

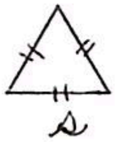
$\xleftrightarrow{\quad - \quad 3.5 \quad + \quad}$

Rel min

The point on $y = \sqrt{x}$ nearest to the point (4,0) is $(3.5, \sqrt{3.5})$

3. A piece of wire 35-m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is a minimum?





$4x + 3s = 35$

$x = \frac{35 - 3s}{4}$

$A = x^2 + \frac{\sqrt{3}}{4}s^2$

$A = \left(\frac{35}{4} - \frac{3s}{4}\right)^2 + \frac{\sqrt{3}}{4}s^2$

$A' = 2\left(\frac{35}{4} - \frac{3s}{4}\right)\left(-\frac{3}{4}\right) + \frac{\sqrt{3}}{2}s$

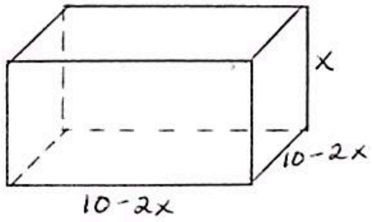
$A' = 0$ when $s = 6.592\ldots$

$\xleftrightarrow{\quad - \quad 6.592\ldots \quad + \quad}$

Rel min

The wire should be cut so that the perimeter of the triangle is 19.776 m & the perimeter of the square is 15.223 m. This will guarantee a minimum area.

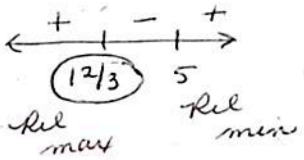
4. A square piece of tin has 10 in on a side. An open box is formed by cutting out equal square pieces x in on a side at the corners and bending upward the projecting portions which remain. Find the maximum volume that can be obtained.



$V = (10-2x)^2 \cdot x$

$v' = (10-2x)^2 \cdot 1 + x \cdot 2(10-2x)(-2)$

$v' = 0$ when $x = 1\frac{2}{3}, \cancel{5}$



The maximum volume that can be attained is

74.674 in^3.

5. If y = 2x-8, what is the minimum value of the product xy?

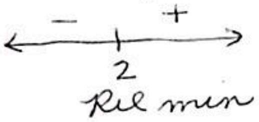
$P = x \cdot y$

$P = x(2x-8)$

$P = 2x^2 - 8x$

$P' = 4x - 8.$

$0 = P'$ when $x = 2$



when $x = 2$, $y = -4$

The minimum product is -8.