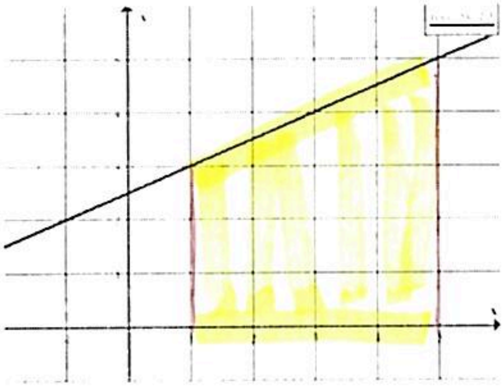
AB Calculus – Riemann Sums Notes and HWK name Key

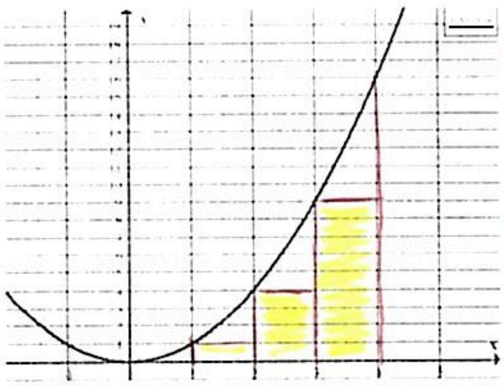
1. Find the area under this function from [1,5]



Trapezoid:

$$\frac{3+5}{2}\cdot 4 = 16$$

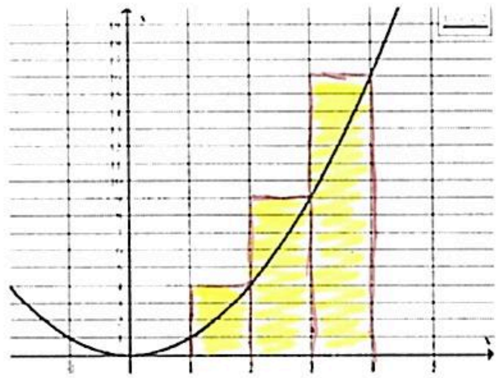
2. Now estimate the area under the curve $y = x^2$ from [1,4]. n=3



$\Delta x = \frac{4-1}{3}$

$\Delta x = 1$

each rectangle $f(c)\cdot\Delta x$



Left Sum – left endpoint

$f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$

$1\cdot 1 + 4\cdot 1 + 9\cdot 1$

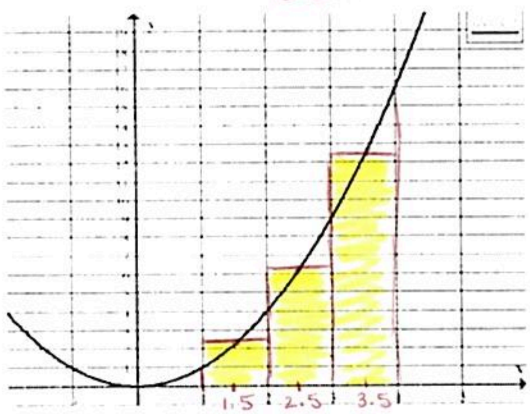
14 → underestimate because f(x) is increasing

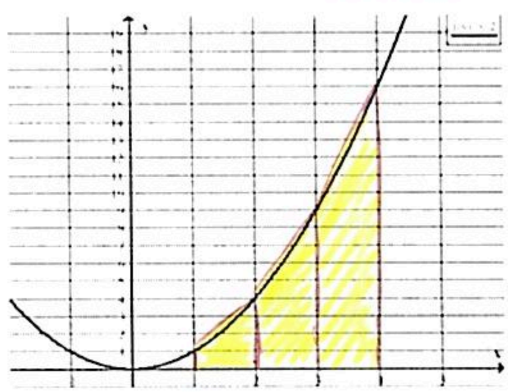
Right Sum – right endpoint

$f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1$

$4\cdot 1 + 9\cdot 1 + 16\cdot 1$

29 → over estimate because f(x) is increasing





AREA OF A TRAPEZOID

$\frac{b_1+b_2}{2}\cdot h$

↓

$\frac{f(c_1)+f(c_2)}{2}\cdot\Delta x$

Midpoint Sum – midpoint of interval

$f(1.5)\cdot 1 + f(2.5)\cdot 1 + f(3.5)\cdot 1$

$2.25\cdot 1 + 6.25\cdot 1 + 12.25\cdot 1$

20.75 → under estimate because f(x) is concave up

Trapezoidal Sum

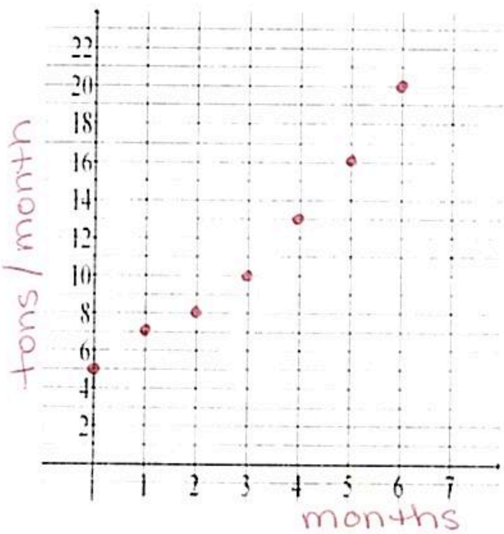
$\frac{f(1)+f(2)}{2}\cdot 1 + \frac{f(2)+f(3)}{2}\cdot 1 + \frac{f(3)+f(4)}{2}\cdot 1$

$\frac{1+4}{2}\cdot 1 + \frac{4+9}{2}\cdot 1 + \frac{9+16}{2}\cdot 1$

21.5 → overestimate because f(x) is concave up

3. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping in the gas:

Time (months)	0	1	2	3	4	5	6
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20



Area = $\frac{\text{tons}}{\text{month}} \cdot \text{months}$

= tons

a. Make an overestimate of the total quantity of pollutants that escaped during the six months use n = 6.

increasing, so right sum $\Delta x = \frac{6-0}{6} = 1$

$7\cdot1 + 8\cdot1 + 10\cdot1 + 13\cdot1 + 16\cdot1 + 20\cdot1$ = 74 tons of pollutants

b. Make an underestimate of the total quantity of pollutants that escaped during the six months use n = 6.
and underestimate

increasing, so left sum

$5\cdot1 + 7\cdot1 + 8\cdot1 + 10\cdot1 + 13\cdot1 + 16\cdot1$ = 59 TONS OF POLLUTANTS

c. Find a midpoint approximation for the amount of gas escaping using 3 equal partitions.

$\Delta x = \frac{6-0}{3} = 2$

$7\cdot2 + 10\cdot2 + 16\cdot2$ = 66 TONS OF POLLUTANTS

d. Find a trapezoidal approximation for the amount of gas escaping using 3 equal partitions.

$\frac{5+8}{2}\cdot2 + \frac{8+13}{2}\cdot2 + \frac{13+20}{2}\cdot2$ = 67 TONS OF POLLUTANTS

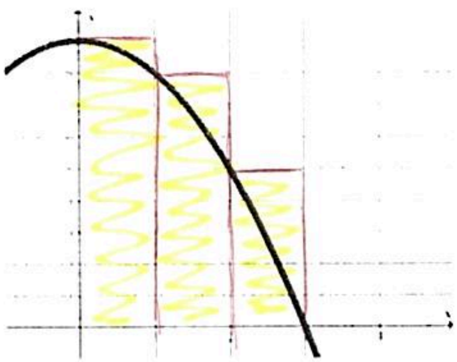
Key

HWK:

1) Find the approximate area under the curve from [0, 3] by using a left and right sum. Use rectangles of width 1.

$\Delta x = \frac{3-0}{3} = 1$

Left Sum

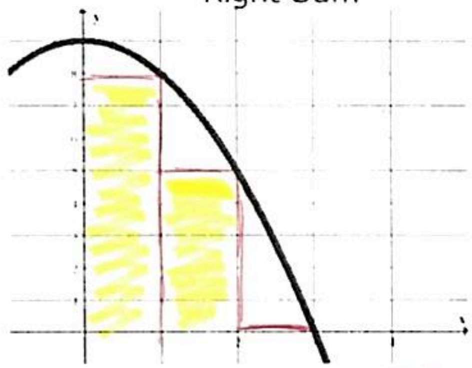


$9 \cdot 1 + 8 \cdot 1 + 5 \cdot 1 = 22$

Is this an overestimate or an underestimate?

How do you know? f(x) is decreasing

Right Sum



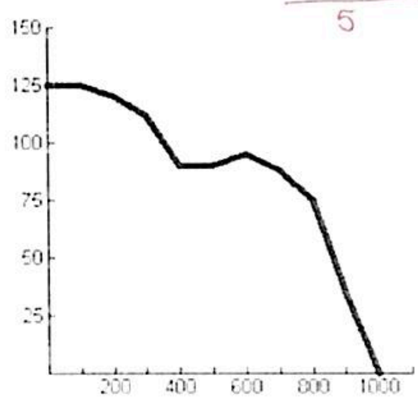
$8 \cdot 1 + 5 \cdot 1 + 0 \cdot 1 = 13$

Is this an overestimate or an underestimate?

How do you know? f(x) is increasing

2) Use the graph and chart to the right.

$\Delta x = \frac{1000-0}{5} = 200$



x	y
0	125
100	125
200	120
300	112
400	90
500	90
600	95
700	88
800	75
900	35
1000	0

a) Estimate the area using right Riemann sums with 5 equal width rectangles.

$120 \cdot 200 + 90 \cdot 200 + 95 \cdot 200 + 75 \cdot 200 + 0 \cdot 2000$

$= 76{,}000$

b) Estimate the area using left Riemann sums with 5 equal width rectangles.

$125 \cdot 200 + 120 \cdot 200 + 90 \cdot 200 + 95 \cdot 200 + 75 \cdot 200$

$= 101{,}000$

c) Estimate the area using midpoint Riemann sums with 5 subintervals of equal length.

$125 \cdot 200 + 112 \cdot 200 + 90 \cdot 200 + 88 \cdot 200 + 35 \cdot 200$

$= 90{,}000$

d) Estimate the area using the Trapezoid Rule with 5 intervals of equal length.

$\frac{125+120}{2} \cdot 200 + \frac{120+90}{2} \cdot 200 + \frac{90+95}{2} \cdot 200 + \frac{95+75}{2} \cdot 200 + \frac{75+0}{2} \cdot 200$

$= 88{,}500$

3) Approximate the area of the region bounded by f(x) from x = 2 to x = 18 using 6 subintervals as indicated in the chart. Assume the function is an increasing function.

3 4 3 2 3 1

x	2	5	9	12	14	17	18
f(x)	0	2	6	9	13	18	21

Subintervals are not = !

a) Left Riemann sum:

$0 \cdot 3 + 2 \cdot 4 + 6 \cdot 3 + 9 \cdot 2 + 13 \cdot 3 + 18 \cdot 1$

101

b) Right Riemann sum:

$2 \cdot 3 + 6 \cdot 4 + 9 \cdot 3 + 13 \cdot 2 + 18 \cdot 3 + 21 \cdot 1$

158

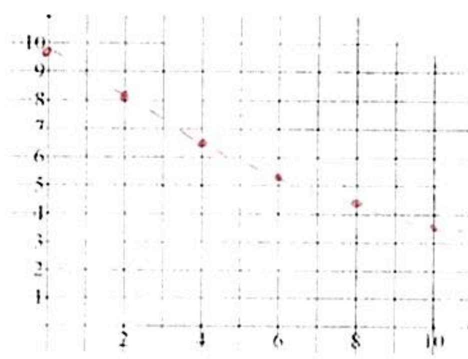
c) Trapezoid sum:

$\frac{0+2}{2} \cdot 3 + \frac{2+6}{2} \cdot 4 + \frac{6+9}{2} \cdot 3 + \frac{9+13}{2} \cdot 2 + \frac{13+18}{2} \cdot 3 + \frac{18+21}{2} \cdot 1$

$= 129.5$

4) You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table below gives your acceleration in m/sec^2 after t seconds.

Time (sec)	0	2	4	6	8	10
Acceleration (m/sec^2)	9.81	8.03	6.53	5.38	4.41	3.61



a. Give upper and lower estimates of your speed at t=10 (use Δt=2).

upper: left

$9.81 \cdot 2 + 8.03 \cdot 2 + 6.53 \cdot 2 + 5.38 \cdot 2 + 4.41 \cdot 2 = 68.32$ m/sec

lower: right

$8.03 \cdot 2 + 6.53 \cdot 2 + 5.38 \cdot 2 + 4.41 \cdot 2 + 3.61 \cdot 2 = 55.92$ m/sec

b. Use the trapezoid method to estimate your speed at t=10. What does the concavity of the graph of acceleration tell you about your estimate?

$\frac{9.81+8.03}{2} \cdot 2 + \frac{8.03+6.53}{2} \cdot 2 + \frac{6.53+5.38}{2} \cdot 2 + \frac{5.38+4.41}{2} \cdot 2 + \frac{4.41+3.61}{2} \cdot 2$

62.12

5) In order to determine the average temperature for the day, a meteorologist decides to record the temperature at eight times during the day. She further decides that these recordings do not have to be equally spaced during the day because she does not need to make several readings during those periods when the temperature is not changing much (as well as not wanting to get up in the middle of the night.) She decides to make one reading at some time during each of the intervals in the table below.

Time	12AM-5AM	5AM-7AM	7AM-9AM	9AM-1PM	1PM-4PM	4PM-7PM	7PM-9PM	9PM-12AM
Temp	42°	57°	72°	84°	89°	75°	66°	52°

a. Using a Riemann sum, write a formula for the average temperature for the day.

$$\frac{5\cdot 42 + 2\cdot 57 + 2\cdot 72 + 4\cdot 84 + 3\cdot 89 + 3\cdot 75 + 2\cdot 66 + 3\cdot 52}{24}$$ degrees

b. Calculate the average temperature.

$\frac{1584}{24} = 66°F$

6) A plane has just crashed six minutes after takeoff. There may be survivors, but you must locate the plane quickly without an extensive search. It was traveling due west and the pilot radioed his speed ever minute from takeoff to the time it crashed. Approximately how far west of the airport did the plane crash? Use a trapezoidal Reimann sum.

~~Minutes~~ hour	0/60	1/60	2/60	3/60	4/60	5/60	6/60
Speed (mph)	90	110	125	135	120	100	70

miles per hour

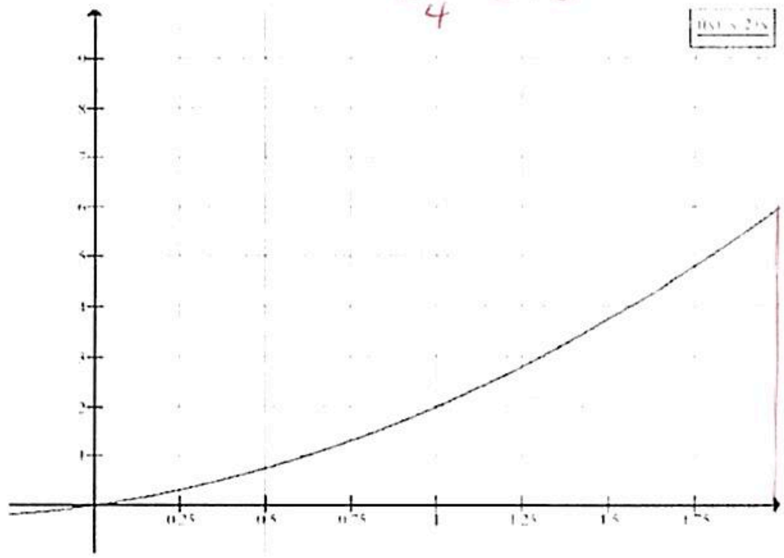
$\Delta x = \frac{1}{60}$

$$\frac{90+110}{2}\cdot\frac{1}{60} + \frac{110+125}{2}\cdot\frac{1}{60} + \frac{125+135}{2}\cdot\frac{1}{60} + \frac{135+120}{2}\cdot\frac{1}{60} + \frac{120+100}{2}\cdot\frac{1}{60} + \frac{100+70}{2}\cdot\frac{1}{60}$$

$11\frac{1}{6}$ miles

7. Let $f(x) = x^2 + x$. Consider the region bounded by the graph of f, the x-axis and the line x=2. Divide the interval [0,2] into 4 equal subintervals. $\Delta x = \frac{2-0}{4} = .5$

x	f(x)
0	0
.25	.3125
.5	.75
.75	1.3125
1	2
1.25	2.8125
1.5	3.75
1.75	4.8125
2	6



a. Obtain a lower estimate for the area of the region by using left endpoints.

$(0)(.5) + (.75)(.5) + (2)(.5) + (3.75)(.5) = 3.25$

b. Obtain an upper estimate by using right endpoints.

$(.75)(.5) + (2)(.5) + (3.75)(.5) + (6)(.5)$

$= 6.25$

c. Find an approximation for the area using trapezoids. Is your estimate too big or too small. Why?

$\frac{0+.75}{2}(.5) + \frac{.75+2}{2}(.5) + \frac{2+3.75}{2}(.5) + \frac{3.75+6}{2}(.5)$

4.75

f(x) is concave up on [0,2]
∴ this is an over estimate.

d. Obtain an estimate for the area using midpoints.

$.3125(.5) + 1.3125(.5) + 2.8125(.5) + 4.8125(.5)$

$= 4.625$