

Precalculus Review

**Domain**

Find the domain and range of the following functions. Write your answers in interval notation. Hint: Use what you know about the functions and then verify your thoughts with the calculator.

1.  $f(x) = x^3 + 3x^2 + 1$

D:  $\mathbb{R}$  R:  $\mathbb{R}$

2.  $f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x+2)(x-2)}$

D:  $x \in \mathbb{R}, x \neq \pm 2$  R:  $y \in \mathbb{R}, y \neq 0, -\frac{1}{4}$

3.  $f(x) = \sqrt{9-x^2}$

D:  $[-3, 3]$   
R:  $[0, 3]$

**Computations**

4. For  $f(x) = x^2 - 2x + 1$ , find

a)  $f(3) = 9 - 6 + 1 = 4$

b)  $f(a+1) = (a+1)^2 - 2(a+1) + 1$

$= a^2 + 2a + 1 - 2a - 2 + 1 = a^2$

c)  $f(x+h) = (x+h)^2 - 2(x+h) + 1$

$= x^2 + 2xh + h^2 - 2x - 2h + 1$

5. If  $f(x) = 3x - 5$  and  $g(x) = x^2 + 2$ , find

a)  $[f \circ g](x) = 3(x^2 + 2) - 5 = 3x^2 + 1$

b)  $[g \circ f](x) = (3x - 5)^2 + 2 = 9x^2 - 30x + 27$

**Linear and Quadratic Functions**

6. Find the equation of the line passing through the points (2,3) and (-5,7). Express your answer in the form  $y - y_1 = m(x - x_1)$ .

$y - y_1 = m(x - x_1)$ .  $m = \frac{7-3}{-5-2} = \frac{4}{7}$

$y - 3 = \frac{4}{7}(x - 2)$

7. Find the equation of the line that passes through the point (-2, 3) and is parallel to the line  $3x - 2y + 6 = 0$ .

$-2y = -3x - 6$   $m = \frac{3}{2}$   
 $y = \frac{3}{2}x + 3$

$y - 3 = \frac{3}{2}(x + 2)$

8. Find the equation of the line that passes through the point (-1, 2) and is perpendicular to the line  $2x - 3y + 5 = 0$ .

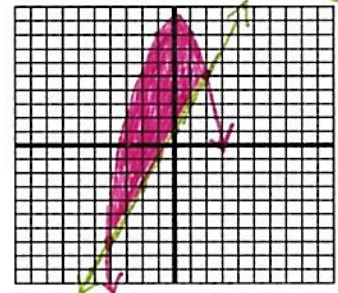
$-3y = -2x - 5$   $\perp m = -\frac{3}{2}$   
 $y = \frac{2}{3}x + \frac{5}{3}$

$y - 2 = -\frac{3}{2}(x + 1)$

9. Find the point of intersection of the lines  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$ .

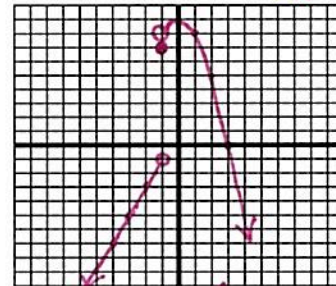
#9  $y = 3x - 7$   
 $x + 5(3x - 7) + 3 = 0$   
 $x + 15x - 35 + 3 = 0$   
 $16x = 32$   
 $x = 2$   
 $(2, 1)$

10. Sketch the region bounded by the curves  $y = 9 - x^2$  and  $y = 2x + 1$ . (bounded means enclosed by)

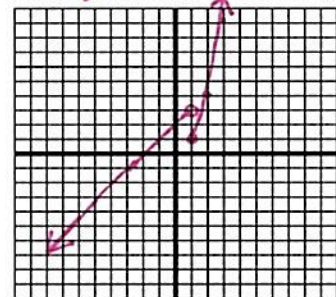


**Piece-Wise Defined Functions**

11. Sketch the graph of  $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 7 & \text{if } x = -1 \\ 9 - x^2 & \text{if } x > -1 \end{cases}$



12. Sketch the graph of  $f(x) = \begin{cases} x + 2 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$



**Factoring**

Factor the following expressions completely.

13.  $x^2 - 64 = (x-8)(x+8)$

14.  $9x - x^3 = x(3-x)(3+x)$

15.  $x^2 + 2x - 3 = (x+3)(x-1)$

16.  $6x^2 - x - 2 = (3x-2)(2x+1)$

17.  $x^2 - 3x - 88 = (x-11)(x+8)$

18.  $2x^3 - 3x^2 - 6x + 9 = x^2(2x-3) - 3(2x-3) = (2x-3)(x^2-3)$

## Simplifying Rational Expressions

$$19. \frac{25-x^2}{x-5} = \frac{(5-x)(5+x)}{x-5} = -(x+5)$$

$$20. \frac{x^2-2x-8}{x^3+x^2-2x} = \frac{(x-4)(x+2)}{x(x+2)(x-1)} = \frac{x-4}{x(x-1)}$$

$$21. \left(\frac{1}{x} - \frac{1}{5}\right) \frac{25x^2}{\left(\frac{1}{x^2} - \frac{1}{25}\right) 25x^2} = \frac{25x-5x^2}{25-x^2} = \frac{5x(5-x)}{(5-x)(5+x)} = \frac{5x}{5+x}$$

## Solving Equations

$$22. 4x - 11 = 3x + 1$$

$$x = 12$$

$$23. (2x-1)(4x-3) = (8x-1)(x+2)$$

$$8x^2 - 10x + 3 = 8x^2 + 15x - 2$$

$$5 = 25x$$

$$x = \frac{1}{5}$$

$$24. (x-2)^2 = 16$$

$$x-2 = \pm 4$$

$$x = 6, -2$$

$$25. (x+1)^3 = -8$$

$$x+1 = -2$$

$$x = -3$$

$$26. \frac{x+3}{5} = \frac{2-x}{7}$$

$$7x+21 = 10-5x$$

$$27. \frac{2x+5}{x+1} = \frac{3}{4}$$

$$8x+20 = 3x+3$$

$$5x = -17$$

## Exponents Simplify. All exponents must be positive.

$$28. x^2(x^4) = x^6$$

$$29. (x^2)^4 = x^8$$

$$30. x^{2/3} \cdot \sqrt[3]{x^2} = x^{2/3} \cdot x^{2/3} = x^{4/3}$$

(RADICAL FORM)

$$31. \frac{(x^3y^{-2})^4}{x^2y^5} = \frac{x^{12}y^{-8}}{x^2y^5} = \frac{x^{10}}{y^{13}}$$

$$x = -\frac{17}{5}$$

## Logarithms Simplify.

$$32. \log_2 8 = 3$$

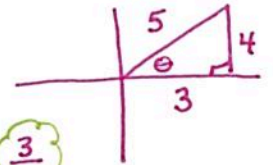
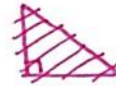
$$33. \ln e^3 = 3$$

$$34. \ln 4 + \ln 5 = \ln 20$$

$$35. \log 10 - \log 5 = \log 2$$

## Trigonometry Review

36. If  $\sin \theta = \frac{4}{5}$ , and  $0 < \theta < \frac{\pi}{2}$ , find (Hint: Draw a triangle.)



a)  $\cos \theta = \frac{3}{5}$

b)  $\tan \theta = \frac{4}{3}$

c)  $\sec \theta = \frac{5}{3}$

d)  $\csc \theta = \frac{5}{4}$

e)  $\cot \theta = \frac{3}{4}$

Find the exact value of each expression. (No calculators)

$$37. \sin \frac{\pi}{6} = \frac{1}{2}$$

$$38. \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$39. \tan \frac{7\pi}{6} = -\frac{1}{\sqrt{3}}$$

$$40. \csc \left(-\frac{5\pi}{6}\right) = -2$$

$$41. \sec \pi = -1$$

$$42. \cot \left(-\frac{3\pi}{2}\right) = 0$$

$$43. \sin \left(\frac{\pi}{2}\right) = 1$$

$$44. \cos \frac{2\pi}{3} = -\frac{1}{2}$$

$$45. \tan \left(\frac{5\pi}{4}\right) = 1$$

Evaluate the following expressions. Because Arcfunctions are restricted, there will only be one answer.

$$46. \cos^{-1} \left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$47. \sin^{-1} \left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$48. \cot^{-1}(-1) = -\frac{\pi}{4}$$

$$49. \tan^{-1}(0) = 0$$

$$50. \sec^{-1}(-2) = \frac{2\pi}{3}$$

$$51. \sec^{-1}(-1) = \pi$$

Solve the following equations for x.

$$52. 2 \cos x = 1 \text{ in the interval } [0, 2\pi)$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$53. 2 \cos^2 x = 1 \text{ in the interval } [0, 2\pi)$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$54. \sin^2 x + \sin x = 0 \text{ in the interval } [0, 2\pi)$$

$$\sin x (\sin x + 1) = 0$$

$$\sin x = 0, -1$$

$$x = 0, \pi, \frac{3\pi}{2}$$

$$55. \cot x = 0 \text{ in the interval } [0, \pi]$$

$$\tan x = \text{und}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Divide using synthetic or long division (if appropriate)

$$56. \frac{x^3 - 4x^2 + 2x + 5}{x-2}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 2 & 5 & \\ & \downarrow & 2 & -4 & -4 & \\ \hline & 1 & -2 & -2 & 1 & \end{array}$$

$$x^2 - 2x - 2 + \frac{1}{x-2}$$

$$57. \frac{3x^3 + 4x + 11}{x^2 - 3x + 2}$$

$$\begin{array}{r} 3x+9 + \frac{25x-1}{x^2-3x+2} \\ \underline{3x^3+0x^2+4x+11} \\ - (3x^3-9x^2+6x) \\ \hline 9x^2-2x+11 \\ - (9x^2-27x+18) \\ \hline 25x-7 \end{array}$$