Notes: Definite Integrals (FTC Part 1)

$\int_a^b f(x)dx$ is called a definite integral. Its answer will be a numeric value.

We can find the value by taking the antiderivative; we will call this F(x), and evaluating F(b)-F(a). For these antiderivatives, we do not need "+C" because they will just cancel out when we subtract.

Example:

$$\int_2^5 (x^2 - 2x)dx = \frac{x^3}{3} - x^2 \Big|_2^5$$

$$= \left(\frac{125}{3} - 25\right) - \left(\frac{8}{3} - 4\right)$$

$$= 39 - 21 = 18$$

Another example:

$$\int_0^\pi 2\cos x dx = 2\sin x \Big|_0^\pi$$

$$= 2\sin\pi - 2\sin 0$$

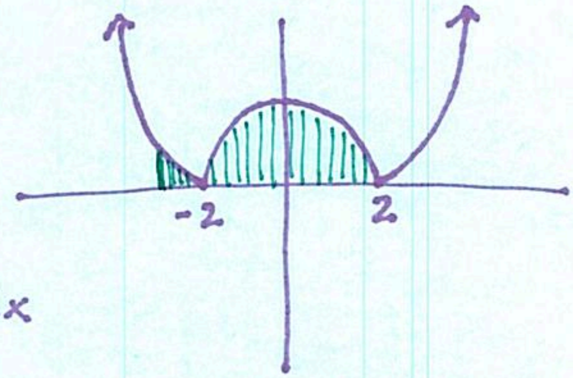
$$= 0$$

Absolute Value: To find the integral of an absolute value function, you must break up the function into its differentiable intervals.

$$\int_{-3}^2 |x^2 - 4| dx = \int_{-3}^{-2} (x^2-4)dx + \int_{-2}^{2} -(x^2-4)dx$$

or

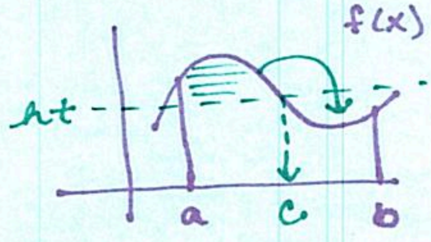
$$= \int_{-3}^{-2} (x^2-4)dx - 2\int_0^2 (x^2-4)dx$$



Average Value of a Function:

If f is integrable on [a,b], its average (mean) value on [a,b] is

$$\frac{1}{b-a}\int_a^b f(x)dx$$



Explanation:

$$\frac{\int_a^b f(x)dx}{b-a} = \frac{area}{width} = f(c) = average\ height\ or\ average\ value$$

c is where the average height occurs.

Example: Find the average value of $f(x) = 3x^2 - 2x$ over [1,3].

$$\frac{1}{3-1}\int_1^3 (3x^2-2x)\,dx = \frac{1}{2}\left(x^3 - x^2 \Big|_1^3\right)$$

$$= \frac{1}{2}\left[(27-9)-(1-1)\right] = 9$$

Mean Value Theorem for Integrals:
If f is continuous on [a,b], then at some point "c" in [a,b]

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

This is an existence theorem. It states that there must be a c on the interval such that f(c) is the average height of the function.

Example:

Using the MVT for integrals, find the value of "c". $f(x) = \frac{4}{x^2}$ *for* [1,4]

1st: find average value

$$\frac{1}{4-1}\int_1^4 4x^{-2}dx$$

$$\frac{1}{3}\left(\frac{-4}{x}\Big|_1^4\right) = \frac{1}{3}(-1+4) = 1$$

2nd: Set $f(x)$ = avg height

$$\frac{4}{x^2} = 1$$

$$x^2 = 4$$

$$x = \pm 2$$

ONLY 2 is on [1, 4]
$\therefore$ c = 2

Example: The rate of water flow out of a pipe, in gallons per hour, can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^3)$. Use Q(t) to approximate the average rate of waterflow out of the pipe in a 24-hour time period.

$$\frac{1}{24}\int_0^{24} Q(t)\,dt$$

use $\boxed{\text{Math}}$ 9

$= -30.531$ $\frac{\text{gallons}}{\text{hour}}$ of water on average flow through the pipe during the 1st 24 hours

Average Rates of Change Problems Name Key

Show all work leading to your answer. There will often be more than one way to solve the problem. You may use a calculator on * problems.

1. Suppose that the velocity function of a particle moving along a coordinate line is $v(t) = 3t^3 + 2$.

a) Find the average velocity over the time interval $1 \le t \le 4$.

$\frac{1}{3}\int_1^4 v(t)\,dt = \frac{1}{3}\left(\frac{3t^4}{4} + 2t \Big|_1^4\right) = \frac{1}{3}\left(3\cdot 64 + 8 - \frac{3}{4} - 2\right) = \frac{263}{4}$ or 65.75

b) Find the average acceleration over the time interval $1 \le t \le 4$.

$\frac{v(4) - v(1)}{4-1} = \frac{194 - 5}{3} = 63$

2. Suppose that the acceleration function of a particle moving along a coordinate line is $a(t) = t + 1$. Find the average acceleration of the particle over the time interval $0 \le t \le 5$.

$\frac{1}{5}\int_0^5 a(t)\,dt = \frac{1}{5}\left(\frac{t^2}{2} + t \Big|_0^5\right) = \frac{1}{5}\left(\frac{25}{2} + 5 - 0\right) = \frac{7}{2}$ or 3.5

*3. During the first 40 seconds of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches of height of $s(t) = \frac{t^3}{\sqrt{10}}$ feet.

a) What is the average height of the rocket during the first 40 seconds?

$\frac{1}{40}\int_0^{40} s(t)\,dt = 5059.644$ ft

b) What is the average velocity of the rocket during the first 40 seconds?

$v(t) = \frac{3}{\sqrt{10}}t^2$

$\frac{1}{40}\int_0^{40} v(t)\,dt = 505.964$ ft/sec

c) What is the average acceleration of the rocket during the first 40 seconds?

$a(t) = \frac{6}{\sqrt{10}}t$

$\frac{1}{40}\int_0^{40} a(t)\,dt = 37.947$ ft/sec^2