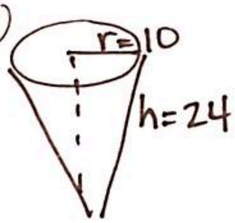
AP Calculus Name Key

Extra Practice—Related Rates

1) A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 cubic feet per minute, how fast is the depth of the water increasing when the water is 16 ft deep?

① 

$\frac{r}{h} = \frac{10}{24}$ $\quad \frac{dV}{dt} = 20\ ft.^3/min$

$24r = 10h$

$r = \frac{10}{24}h \rightarrow \boxed{\frac{5}{12}h}$

② $\left.\frac{dh}{dt}\right|_{h=16}$

③ $V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

$= \frac{1}{3}\pi\frac{25}{144}h^2 \cdot h \rightarrow \cdots \rightarrow \frac{25}{432}h^3 = V$

④ $V' = \frac{75}{432}h^2\frac{dh}{dt}$

$20 = \frac{75}{432}(16)^2\pi\frac{dh}{dt}$

$20 = \frac{25}{144}(16)^2\pi\frac{dh}{dt}$

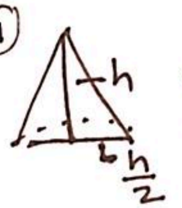
$20 = \frac{400}{9}\pi\frac{dh}{dt}$

$20 = \frac{400\pi}{9}\frac{dh}{dt}$

$\frac{9}{20\pi} = \quad ft./min = \frac{dh}{dt}$

$\boxed{\left.\frac{dh}{dt}\right|_{h=16} \frac{9}{20\pi}\ ft./min.}$

2) Sand pouring from a chute forms a conical pile whose height and diameter are always the same. If the height increases at a constant rate of 5 ft/min, at what rate is sand pouring from the chute when the pile is 10 ft high?

① 

$\frac{dh}{dt} = 5ft/min$

$r = \frac{1}{2}h$

② $\left.\frac{dV}{dt}\right|_{h=10}$

$\llcorner$ use in terms of h

③ $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$

$= \frac{1}{3}\pi\frac{1}{4}h^3$

$= \frac{\pi}{12}h^3$

④ $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}(10^2)\cancel{\frac{dh}{dt}}(5)$

$\frac{dV}{dt} = \frac{500\pi}{4}\cancel{\frac{dh}{dt}}$

$\boxed{\left.\frac{dV}{dt}\right|_{h=10} 125\pi\ ft./min}$

1. Wheat is falling from a chute onto a level floor at a rate of 8π ft^3/min to form a conical pile. If the height of the pile is always equal to the radius of its base, at what rate is the radius increasing when the pile is 8 feet deep?

①  $\frac{dV}{dt} = 8\pi$ ft.^{3}/m $\quad h = r$

② $\left.\frac{dr}{dt}\right|_{r=8}$

③ $\frac{1}{3}\pi r^2 \cdot h = V$

$V = \frac{1}{3}\pi r^2 \cdot r$

$V = \frac{1}{3}\pi r^3$

④ $\frac{dV}{dt} = \pi r^2 \frac{dr}{dt}$

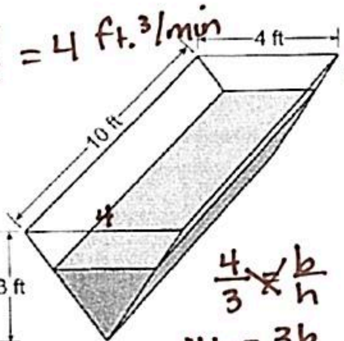
⑤ $8\pi = \pi(8)^2 \frac{dr}{dt}$

$\frac{1}{8} = \frac{dr}{dt}$

⑥ $\boxed{\left.\frac{dV}{dt}\right|_{r=8} \ \frac{1}{8} \text{ ft./min}}$

2. The watering trough in the diagram below is being filled at a rate of $\boxed{\text{4 cubic feet}}$ of water per minute. How fast is the depth of the water, *h*, increasing when the trough is half-full by volume.

$\frac{dV}{dt} = 4$ ft.^{3}/min



1.

$\frac{4}{3} = \frac{b}{h}$

$\frac{4h}{4} = \frac{3b}{4}$

$\boxed{b = \frac{4}{3}h}$

2. $\left.\frac{dh}{dt}\right|_{V=30}$

3. $V = B \cdot h$

$V = \frac{1}{2} b \cdot h \cdot 10$

$V = \frac{1}{2}\left(\frac{4}{3}h\right)h \cdot 10$

$V = 5\left(\frac{4}{3}h^2\right) \rightarrow \frac{20}{3}h^2$

4. $\frac{dV}{dt} = \frac{40}{3} h \frac{dh}{dt}$

⑤ $4 = \frac{40}{3} h \frac{dh}{dt}$

$4 = \frac{40}{3}(\sqrt{4.5})\frac{dh}{dt}$

$\frac{120}{40}$

$\frac{.3}{\sqrt{4.5}} = \frac{dh}{dt}$

$\frac{dh}{dt} = .141$

$\boxed{\left.\frac{dh}{dt}\right|_{V=30} \ .141 \text{ ft./min}}$

For h

?

$\frac{1}{2} b \cdot h \cdot 10 = 30$

$\frac{1}{2}\left(\frac{4}{3}h\right)h \cdot 10 =$

$5\left(\frac{4}{3}\right)h^2 =$

$\frac{20}{3}h^2 = 3$

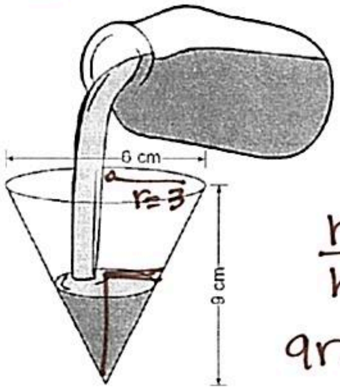
$h^2 = \frac{9(}{2}$

$h = \sqrt{\frac{90}{20}}$

$h = \sqrt{4.5}$

3. A conical-shaped paper cup is shown in the diagram below. If water is being poured into the cup at a rate of 1 cubic centimeter per second, how fast is the depth of the water increasing when the water is 4 centimeters deep?

①



$\frac{dV}{dt}$ 1 cm^3/sec

$\frac{r}{h} = \frac{3}{9}$

$9r = 3h$

$r = \frac{1}{3}h$

② $\left.\frac{dh}{dt}\right|_{h=4}$

③ $V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 \cdot h$

$V = \frac{\pi h^3}{27}$

$V = \frac{\pi h^3}{27}$

$V = \frac{\pi h^3}{27}$

④ $\frac{dV}{dt} = \frac{3h^2}{27}\pi \frac{dh}{dt}$

⑤ $1 = \frac{h^2 16}{9}\pi \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{9}{16\pi} \text{ cm/sec}}$

$\boxed{\frac{dh}{dt} = .179 \text{ cm/sec}}$

⑥ $\boxed{\left.\frac{dh}{dt}\right|_{h=4} \ .179 \text{ cm/sec}}$

or

$\frac{dh}{dt} = \frac{9}{16\pi}$ cm/sec