Key

1) $X = $ Number 1
$Y = $ Number 2

$\underbrace{P = XY^2}_{\text{Trying to maximize}}$ $\quad X + Y = 9$

$\underbrace{X = 9 - Y}$

$P = (9-Y)Y^2$

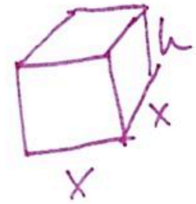
$P = 9Y^2 - Y^3$

$P' = 18Y - 3Y^2$

$0 = 18Y - 3Y^2$

$0 = 3Y(6 - Y)$

$\boxed{Y = 6 \quad X = 3}$

2) 

$\underbrace{48 = x^2 + 4xh}_{\text{Constraint}}$

$\underbrace{V = x^2 h}_{\text{Trying to maximize}}$

$48 - x^2 = 4xh$

$\frac{48 - x^2}{4x} = h$

$12x^{-1} - \frac{x}{4} = h$

$V = x^2\left(12x^{-1} - \frac{x}{4}\right)$

$V = 12x - \frac{1}{4}x^3$

$V' = 12 - \frac{3}{4}x^2$

$V' = 0 \text{ @ } x = 4$

(Plug in to constraint to solve for h)

Dimensions of Box - 4 ft x 4 ft x 2 ft

3) 

No top

$SA = 2\pi rh + \pi r^2$

$\underbrace{3\pi = 2\pi rh + \pi r^2}_{\text{Constraint}}$

$\underbrace{V = \pi r^2 h}_{\text{Trying to maximize}}$

$h = \frac{3\pi - \pi r^2}{2\pi r}$

$V = \cancel{\pi} r^{\cancel{2}}\left(\frac{3\pi - \pi r^2}{\cancel{2\pi r}}\right)$

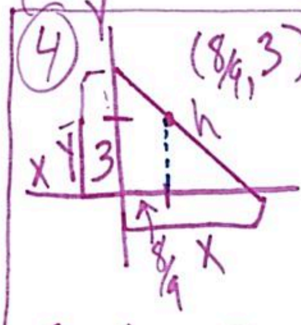
$V = \frac{3\pi r}{2} - \frac{\pi r^3}{2}$

$V' = \frac{3\pi}{2} - \frac{3\pi r^2}{2}$

$V' = 0 \text{ @ } r = 1$

$r = 1$ ft
$h = 1$ ft

(Plug into Constraint to solve for h)

4) 

$x^2 + y^2 = h^2$

$h = \sqrt{x^2 + y^2}$ ← Trying to minimi

Similar Triangles

$\frac{X}{X - 8/9} = \frac{Y}{3}$

$Y = \frac{3X}{X - 8/9}$

$Y = \frac{27X}{9X - 8}$

$h = \sqrt{x^2 + \left(\frac{27x}{9x-8}\right)^2}$

$h = \left(x^2 + \left(\frac{27x}{9x-8}\right)^2\right)^{1/2}$

$h' = \frac{1}{2}\left(x^2 + \left(\frac{27x}{9x-8}\right)^2\right)^{-1/2} \cdot$

$\left[2x + 2\left(\frac{27x}{9x-8}\right)^1 \cdot \left(\frac{(9x-8)(27) - 27x \cdot}{(9x-8)^2}\right.\right.$

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4 (continued)

$$h' = \frac{1}{2}\left(x^2 + \left(\frac{27x}{9x-8}\right)^2\right)^{-1/2} \cdot \left[2x + 2\left(\frac{27x}{9x-8}\right)^1 \cdot \left(\frac{(9x-8)(27) - 27x \cdot 9}{(9x-8)^2}\right)\right]$$

$h' = 0$ @ $\boxed{\begin{array}{l} x = 2.888 \\ y = 4.333 \\ h = 5.208 \end{array}}$

(Plug in to original eqn to solve for h+y)

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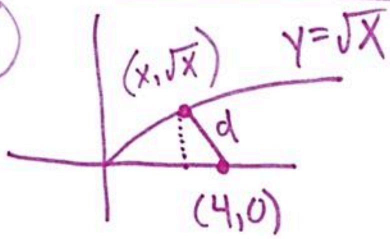

$D = \sqrt{(4-x)^2 + (0-\sqrt{x})^2}$ ← Trying to minimize (nearest point)

$D = \sqrt{16 - 8x + x^2 + x}$

$D = \sqrt{x^2 - 7x + 16}$

$D = (x^2 - 7x + 16)^{1/2}$

$D' = \frac{1}{2}(x^2 - 7x + 16)^{-1/2}(2x - 7)$

$D' = 0$ @ $x = 3.5$

Point is $(3.5, \sqrt{3.5})$ or $(3.5, 1.870)$

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$V = 20\pi$; $V = \pi r^2 h$

$20\cancel{\pi} = \cancel{\pi} r^2 h$ — Constr

$20 = r^2 h$

$h = 20/r^2$

$\underbrace{\text{Cost} = 10(2\pi r^2) + 8(2\pi r h)}_{\text{Trying to minimize}}$

$\text{Cost} = 20\pi r^2 + 16\pi r\left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + 320\pi r^{-1}$

$C' = 40\pi r - 320\pi r^{-2}$

$C' = 0$ @ $\boxed{\begin{array}{l} r = 2\text{ m} \\ h = 5\text{ m} \end{array}}$ Cost = \$753.9

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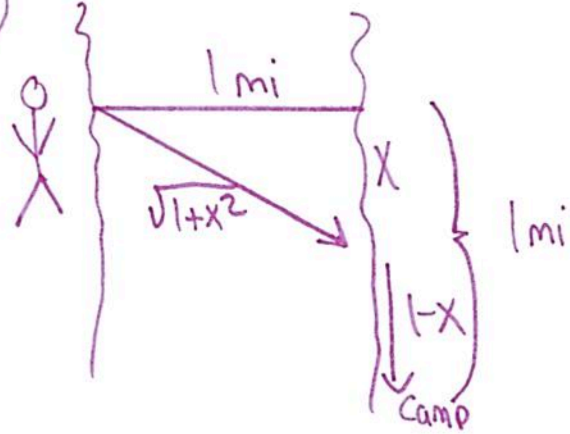

$Rate_w = 3$ mph

$Rate_s = 2$ mph

$D_T = r_T t_T$

$t_T = \frac{D_T}{r_T}$ ← Trying to minimize

$$t_T = \frac{D_w}{R_w} + \frac{D_s}{R_s}$$

$$t_T = \frac{1-x}{3} + \frac{\sqrt{1+x^2}}{2}$$

$$t_T = \frac{1}{3} - \frac{1}{3}x + \frac{1}{2}(1+x^2)^{1/2}$$

$$t_T' = \frac{-1}{3} + \frac{1}{4}(1+x^2)^{-1/2}(2x)$$

$$t_T' = 0 \text{ @ } \underline{x = .894}$$

What route minimizes time?

D walking: $1-x = .105$ mi

D Swim: $\sqrt{1+x^2} = 1.341$ mi

Time = .706 hours ≈ 42.36 min

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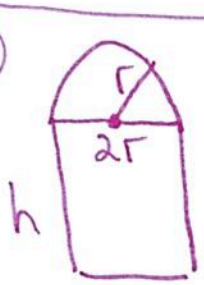

Perimeter = $\pi r + 2h + 2r$

$\underbrace{12 = \pi r + 2h + 2r}_{\text{Constraint}}$

$$h = \frac{12 - \pi r - 2r}{2}$$

$$h = 6 - \tfrac{1}{2}\pi r - r$$

$A = 2rh$

$A = 2r(6 - \tfrac{1}{2}\pi r - r)$

$A = 12r - \pi r^2 - 2r^2$

$A' = 12 - 2\pi r - 4r$

$A' = 0$ @ $r = 1.166$ ft $h = 3$ ft

$A = 7.001$ ft^2