

Finding Derivatives using a Table

Two functions, f and g , are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are shown in the table.

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Based on the table, find the following derivatives:

1) $\frac{d}{dx}(f(x) + g(x))$, evaluated at $x = 4$.

$$\begin{aligned}
 h(x) &= f(x) + g(x) \\
 &= f'(x) + g'(x) \\
 &= f'(4) + g'(4) \\
 &= -\frac{4}{5} + -\frac{1}{3} \\
 &= \frac{-12}{15} + \frac{-5}{15} = \boxed{\frac{-17}{15}}
 \end{aligned}$$

3) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$, evaluated at $x = 0$.

$$\begin{aligned}
 &= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \\
 &= \frac{g(0)f'(0) - f(0)g'(0)}{[g(0)]^2} \\
 &= \frac{(-2)\left(\frac{3}{2}\right) - \left(\frac{1}{2}\right)(-1)}{(-2)^2} \\
 &= \frac{-3 + \frac{1}{2}}{4} = \frac{-2.5}{4} = -\frac{5}{2} \cdot \frac{1}{4} = \boxed{\frac{-5}{8}}
 \end{aligned}$$

2) $\frac{d}{dx}(f(x) \cdot g(x))$, evaluated at $x = 1$.

$$\begin{aligned}
 h(x) &= f(x) \cdot g(x) \\
 &= f(x)g'(x) + g(x)f'(x) \\
 &= f(1)g'(1) + g(1)f'(1) \\
 &= \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + (1)\left(\frac{5}{3}\right) \\
 &= \frac{2}{9} + \frac{5}{3} \\
 &= \frac{2}{9} + \frac{15}{9} = \boxed{\frac{17}{9}}
 \end{aligned}$$

4) $\frac{d}{dx}(f(g(x)))$, evaluated at $x = 3$.

$$\begin{aligned}
 &= f'(g(x)) \cdot g'(x) \quad \left. \begin{array}{l} \text{shell: } f(x) \} f'(x) \\ \text{choc. } g(x) \} g'(x) \end{array} \right\} f'(g(x))g'(x) \\
 &= f'(g(3))g'(3) \\
 &= f'(2) \cdot (-3) \\
 &= \frac{1}{4} \cdot (-3) \\
 &= \boxed{\frac{-3}{4}}
 \end{aligned}$$

Using the table, evaluate each of the following at $x = -1$.

x	f	f'	g	g'
-1	4	7	5	2
4	-2	3	-1	π

$$5) \frac{d}{dx}(f+g)$$

$$f'(x) + g'(x)$$

$$f'(-1) + g'(-1)$$

$$7 + 2$$

$$\boxed{9}$$

$$6) \frac{d}{dx}\left(\frac{f}{g}\right)$$

$$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{g(-1)f'(-1) - f(-1)g'(-1)}{[g(-1)]^2}$$

$$\frac{(5)(7) - (4)(2)}{25} = \frac{35-8}{25} = \frac{27}{25}$$

$$\boxed{\frac{27}{25}}$$

$$7) \frac{d}{dx}(f \cdot g)$$

$$f(x)g'(x) + f'(x)g(x)$$

$$f(-1)g'(-1) + f'(-1)g(-1)$$

$$(4)(2) + (7)(5)$$

$$8 + 35$$

$$\boxed{43}$$

$$8) \frac{d}{dx}((f)^2)$$

$$2f(x)f'(x)$$

$$2f(-1)f'(-1)$$

$$2(4)(7) = \boxed{56}$$

$$9) \frac{d}{dx}(\sqrt{f+g}) \quad (f(x)+g(x))^{\frac{1}{2}}$$

$$\frac{1}{2} [f'(x)+g'(x)] (f(x)+g(x))^{-\frac{1}{2}}$$

$$\frac{f'(x)+g'(x)}{2\sqrt{f(x)+g(x)}}$$

$$\frac{f'(-1)+g'(-1)}{2\sqrt{f(-1)+g(-1)}}$$

$$\frac{7+2}{2\sqrt{4+5}} = \frac{9}{2 \cdot 3} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

$$10) \frac{d}{dx}\left(\frac{1}{f}\right) [f(x)]^{-1}$$

$$-1 \cdot f'(x) [f(x)]^{-2}$$

$$\frac{-1 \cdot f'(x)}{[f(x)]^2}$$

$$\frac{-1 \cdot f'(-1)}{[f(-1)]^2}$$

$$\frac{-1(7)}{4^2} = \boxed{-\frac{7}{16}}$$