Limit Definition of a Derivative name Key

1. $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$ cosx

$f(x) = \sin x$
$f'(x) = \cos x$

2. $\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - 3 - (x^2 - 2x - 3)}{h} =$ $2x - 2$

$f(x) = x^2 - 2x - 3$
$f'(x) = 2x - 2$

3. $\lim_{h \to 0} \frac{2(x+h)^2 - (x+h) + 3 - (2x^2 - x + 3)}{h} =$ $4x - 1$

$f(x) = 2x^2 - x + 3$
$f'(x) = 4x - 1$

4. $\lim_{h \to 0} \frac{2(2+h)^2 - (2+h) + 3 - 2(2)^2 + 2 - 3}{h} =$ 7

$f(x) = 2x^2 - x + 3$
$f'(x) = 4x - 1$
$f'(2) = 4(2) - 1 = 7$

5. $\lim_{h \to 0} \frac{\cos(\pi + h) - \cos \pi}{h} =$ 0

$f(x) = \cos x$
$f'(x) = -\sin x$
$f'(\pi) = -\sin \pi = 0$

6. Use the limit definition to find the derivative of f(x) =x^2 – 2x

$$f' = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$f' = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-2x} - 2h \cancel{-x^2} + \cancel{2x}}{h}$$

$$f' = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = 2x - 2$$

7. Use the limit definition to find the slope of the tangent line of f(X) at x=3.

$f(x) = \frac{1}{2x-3}$

$$\lim_{x \to 3} \frac{\frac{1}{2x-3} - \frac{1}{3}}{x - 3} = f'(3)$$

$$f'(3) = \lim_{x \to 3} \left[\left(\frac{3 - (2x-3)}{3(2x-3)} \right) \cdot \frac{1}{x-3} \right]$$

$$f'(3) = \lim_{x \to 3} \left[\frac{-2x+6}{3(2x-3)} \cdot \frac{1}{x-3} \right] = \lim_{x \to 3} \left[\frac{-2\cancel{(x-3)}}{3(2x-3)} \cdot \frac{1}{\cancel{x-3}} \right] = \frac{-2}{9}$$