

$$1) \quad x^2 + y^2 = 9$$

$$\cancel{2x} + 2y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{2y}{2y} \left(\frac{dy}{dx} \right) = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$7) \quad x^3 y^3 - y = x$$

$$x^3 \cdot 3y^2 \left(\frac{dy}{dx} \right) + y^3 \cdot 3x^2 - \left(\frac{dy}{dx} \right) = 1$$

$$3x^3 y^2 \left(\frac{dy}{dx} \right) - \left(\frac{dy}{dx} \right) = 1 - 3x^2 y^3$$

$$\frac{dy}{dx} \left(3x^3 y^2 - 1 \right) = 1 - 3x^2 y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2 y^3}{3x^3 y^2 - 1}$$

$$5) \quad x^3 - xy + y^2 = 7$$

$$\cancel{x^3} - x \left(\frac{dy}{dx} \right) - \cancel{y} + 2y \left(\frac{dy}{dx} \right) = 0$$

$$-x \left(\frac{dy}{dx} \right) + 2y \left(\frac{dy}{dx} \right) = y - x^3$$

$$\frac{dy}{dx} (-x + 2y) = y - x^3$$

$$\frac{dy}{dx} = \frac{y - x^3}{2y - x}$$

$$9) \quad x e^y - 10x + 3y = 0$$

$$x \cdot e^y \left(\frac{dy}{dx} \right) + e^y - 10 + 3 \left(\frac{dy}{dx} \right) = 0$$

$$x e^y \left(\frac{dy}{dx} \right) + 3 \left(\frac{dy}{dx} \right) = 10 - e^y$$

$$\frac{dy}{dx} (x e^y + 3) = 10 - e^y$$

$$\frac{dy}{dx} = \frac{10 - e^y}{x e^y + 3}$$

$$11) \sin x + 2 \cos 2y = 1$$

$$\cos x - 4 \sin 2y \left(\frac{dy}{dx} \right) = 0$$

$$-4 \sin 2y \left(\frac{dy}{dx} \right) = -\cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{4 \sin 2y}$$

$$55) \ x^2 - y^2 = 36$$

$$2x - 2y \left(\frac{dy}{dx} \right) = 0$$

$$-2y \left(\frac{dy}{dx} \right) = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(y - \frac{x^2}{y} \right) y}{(y^2)y}$$

$$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}$$

$$17) \ x^2 - 3 \ln y + y^2 = 10$$

$$2x - \frac{3}{y} \left(\frac{dy}{dx} \right) + 2y \left(\frac{dy}{dx} \right) = 0$$

$$-\frac{3}{y} \left(\frac{dy}{dx} \right) + 2y \left(\frac{dy}{dx} \right) = -2x$$

$$\frac{dy}{dx} \left(-\frac{3}{y} + 2y \right) = -2x$$

$$\frac{dy}{dx} = \frac{(-2x)y}{\left(-\frac{3}{y} + 2y \right)y}$$

$$\frac{dy}{dx} = \frac{-2xy}{-3 + 2y^2}$$

$$57) \ y^2 = x^3$$

$$2y \left(\frac{dy}{dx} \right) = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\frac{d^2y}{dx^2} = \frac{2y \cdot 6x - 3x^2 \cdot 2 \left(\frac{dy}{dx} \right)}{(2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(12xy - 6x^2 \left(\frac{3x^2}{2y} \right) \right) 2y}{(4y^2) 2y}$$

$$\frac{d^2y}{dx^2} = \frac{24xy^2 - 18x^4}{8y^3}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - 9x^4}{4y^3}$$

$$67) y = \frac{x^2 \sqrt{3x-2}}{(x+1)^2}$$

$$\ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x+1} \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{(x+1)} \right] \frac{x^2 \sqrt{3x-2}}{(x+1)^2}$$

< you can
stop here!

$$\frac{dy}{dx} = \frac{2\sqrt{3x-2}}{(x+1)^2} + \frac{3x^2}{2\sqrt{3x-2}} - \frac{2x^2\sqrt{3x-2}}{(x+1)^3}$$

$$65) y = x\sqrt{x^2+1} = x(x^2+1)^{\frac{1}{2}}$$

$$\ln y = \ln x + \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2+1}$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \left[\frac{1}{x} + \frac{x}{x^2+1} \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{x^2+1} \right] x\sqrt{x^2+1} \quad < \text{you can stop here}$$

$$\frac{dy}{dx} = \frac{\cancel{x^2+1}}{\cancel{x^2+1}-1} \cdot \frac{\sqrt{x^2+1}}{1} + \frac{x^2}{\sqrt{x^2+1}}$$

$$\frac{dy}{dx} = \frac{x^2+1+x^2}{\sqrt{x^2+1}} = \frac{2x^2+1}{\sqrt{x^2+1}}$$

$$71) \quad y = x^{\frac{2}{x}}$$

$$\ln y = \frac{2}{x} \ln x$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{2}{x} \cdot \frac{1}{x} + \ln x \cdot \frac{-2}{x^2}$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \left[\frac{2}{x^2} - \frac{2 \ln x}{x^2} \right] x^{\frac{2}{x}}$$

$$\frac{dy}{dx} = \left(\frac{2 - 2 \ln x}{x^2} \right) x^{\frac{2}{x}} \quad \leftarrow \text{you can stop here!}$$

$$\frac{dy}{dx} = \frac{2 - 2 \ln x}{x^{2 - 2/x}}$$

$$73) \quad y = (x-2)^{x+1}$$

$$\ln y = (x+1) \ln(x-2)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \left[(x+1) \cdot \frac{1}{x-2} + \ln(x-2) \cdot 1 \right] \cdot y$$

$$\frac{dy}{dx} = \left[\frac{x+1}{x-2} + \ln(x-2) \right] (x-2)^{x+1} \quad \leftarrow \text{you can stop here!}$$

$$\frac{dy}{dx} = (x+1)(x-2)^x + (x+2)^{x+1} \cdot \ln(x-2)$$

83) → answers vary