# Key to U-sub Day 2

1. $\int x\sqrt{x+3}\,dx = \int (u-3)\,u^{1/2}\,du$

$u = x+3 \qquad = \int \left(u^{3/2} - 3u^{1/2}\right)du$

$\frac{du}{dx} = 1 \qquad = \frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C$

$dx = du$

$x = u-3 \qquad = \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

2. $\int_1^2 2x^2\sqrt{x^3+1}\,dx = \int_2^9 2x^2\cdot u^{1/2}\cdot\frac{du}{3x^2}$

$u = x^3+1$

$\frac{du}{dx} = 3x^2 \qquad = \frac{2}{3}\int_2^9 u^{1/2}\,du$

$dx = \frac{du}{3x^2} \qquad = \frac{2}{3}\cdot\frac{2}{3}u^{3/2}\Big|_2^9$

$= \frac{4}{9}(27) - \frac{4}{9}\sqrt{8}$

$= 12 - \frac{8}{9}\sqrt{2}$

3. $\int_{\frac{\pi}{15}}^{\frac{\pi}{10}} \cos 5x\,dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos u\,\frac{du}{5}$

$u = 5x \qquad = \frac{1}{5}\sin u\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5} \qquad = \frac{1}{5}(1) - \frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1-\sqrt{3}}{10}$

#20 $\int a(t) = \int(6t-2)dt \qquad v(3) = 25$

$s(1) = 10$

$v(t) = 3t^2 - 2t + C_1$

$25 = 27 - 6 + C_1$

$4 = C_1$

$\int v(t) = \int(3t^2 - 2t + 4)dt$

C

$s(t) = t^3 - t^2 + 4t + C_2$

$10 = 1 - 1 + 4 + C_2$

$C_2 = 6$

4. $\int x^2\sqrt{x-5}\,dx = \int (u+5)^2\cdot u^{1/2}\,du$

$u = x-5$

$\frac{du}{dx} = 1$

$dx = du$

$x = u+5$

$= \int (u^2+10u+25)u^{1/2}\,du$

$= \int \left(u^{5/2} + 10u^{3/2} + 25u^{1/2}\right)du$

$= \frac{2}{7}u^{7/2} + 10\cdot\frac{2}{5}u^{5/2} + 25\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{7}(x-5)^{7/2} + 4(x-5)^{5/2} + \frac{50}{3}(x-5)^{3/2} + C$

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5. $\int \frac{x}{\sqrt[3]{x+5}}\,dx = \int \frac{u-5}{u^{1/3}}\,du$

$u = x+5$

$\frac{du}{dx} = 1$

$dx = du$

$x = u-5$

$= \int \left(u^{2/3} - 5u^{-1/3}\right)du$

$= \frac{3}{5}u^{5/3} - 5\cdot\frac{3}{2}u^{2/3} + C$

$= \frac{3}{5}(x+5)^{5/3} - \frac{15}{2}(x+5)^{2/3} + C$

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6. $\int_1^2 (5x-4+7x^{-2})\,dx = \frac{5x^2}{2} - 4x - 7x^{-1}\Big|_1^2$

$= \left(5\cdot 2 - 8 - \frac{7}{2}\right) - \left(\frac{5}{2} - 4 - 7\right)$

$= 2+4+7-6$

$= 7$

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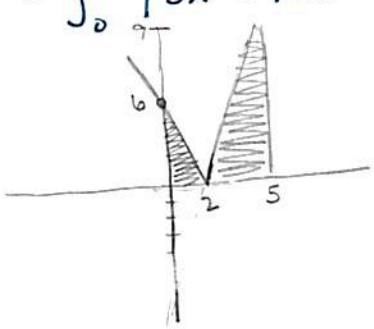
7. $\int_0^5 |3x-6|\,dx = \frac{6\cdot 2}{2} + \frac{9\cdot 3}{2}$




$= \frac{29}{2}$

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8. $\int \frac{1}{3x+4}\,dx = \int \frac{1}{u}\cdot\frac{du}{3}$

$u = 3x+4$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|3x+4| + C$

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9. $\int \sin x\, e^{\cos x}\,dx = \int \sin x\cdot e^u\,\frac{du}{-\sin x}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$= -\int e^u\,du$

$= -e^u + C$

$= -e^{\cos x} + C$

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10. $\int_e^{e^2} \frac{(\ln x)^4}{x}\,dx = \int_1^2 \frac{u^4}{x}\,x\,du$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$= \int_1^2 u^4\,du$

$= \frac{u^5}{5}\Big|_1^2$

$= \frac{32}{5} - \frac{1}{5} = \frac{31}{5}$

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11. $\int \frac{e^{2x}}{1+e^{2x}}\,dx = \int \frac{e^{2x}}{u}\cdot\frac{du}{2e^{2x}}$

$u = 1+e^{2x}$

$\frac{du}{dx} = 2e^{2x}$

$dx = \frac{du}{2e^{2x}}$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln(1+e^{2x}) + C$

2. $\int \tan 3x\, dx = \int \tan u \cdot \frac{du}{3}$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$= -\frac{1}{3} \ln|\cos u| + C$

$= -\frac{1}{3} \ln|\cos 3x| + C$

3. $\int 5 \sec(3x)dx = 5\int \sec u \cdot \frac{du}{3}$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$= \frac{5}{3} \ln|\sec u + \tan u| + C$

$= \frac{5}{3} \ln|\sec 3x + \tan 3x| + C$

14. $\int \frac{\sec^2 3x}{\tan 3x} dx = \int \frac{\sec^2 3x}{u} \cdot \frac{du}{3\sec^2 3x}$

$u = \tan 3x$

$\frac{du}{dx} = 3\sec^2 3x$

$dx = \frac{du}{3\sec^2 3x}$

$= \frac{1}{3}\int \frac{1}{u}\, du$

$= \frac{1}{3} \ln|u| + C$

$= \frac{1}{3} \ln|\tan 3x| + C$

15. $\int f''(x)dx = \int 3x^2 dx$  $f'(0) = 5$

$f(0) = -2$

$f'(x) = x^3 + C_1$

$5 = C_1$

$\int f'(x)\,dx = \int (x^3 + 5)\,dx$

$f(x) = \frac{x^4}{4} + 5x + C_2$

$-2 = C_2$

$f(x) = \frac{x^4}{4} + 5x - 2$

16. $y = \frac{x^3}{\sec x}$  $y' = x^3(-\sin x) + \cos x(3x^2)$

$y = x^3 \cos x$

$y' = -x^3 \sin x + 3x^2 \cos x$

17. $y = \sin^3(x^7)$

| | |
|---|---|
| S: $(\quad)^3$ | $3(\quad)^2$ |
| C: $\sin(\quad)$ | $\cos(\quad)$ |
| P: $x^7$ | $7x^6$ |

$y' = 21x^6 \cos(x^7)\sin^2(x^7)$

18. $f(x) = \sqrt{x-1}$

$(f^{-1})'(3) = \frac{1}{f'(10)}$

$3 = \sqrt{x-1}$

$9 = x - 1$

$x = 10$

$f(10) = 3$

$f' = \frac{1}{2}(x-1)^{-1/2}$

$f' = \frac{1}{2\sqrt{x-1}}$

$f'(10) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

$(f^{-1})'(3) = 6$

19. $\int f''(x) = \int (\cos x + e^{5x})dx$  $f(0) = \frac{4}{25}$

$f'(0) = \frac{3}{5}$

$f'(x) = \sin x + \frac{1}{5}e^{5x} + C_1$

$\frac{3}{5} = \frac{1}{5} + C_1$

$C_1 = \frac{2}{5}$

$\int f'(x) = \int (\sin x + \frac{1}{5}e^{5x} + \frac{2}{5})dx$

$f(x) = -\cos x + \frac{1}{25}e^{5x} + \frac{2}{5}x + C_2$

$\frac{4}{25} = -1 + \frac{1}{25} + C_2$

$\frac{4}{25} + \frac{25}{25} - \frac{1}{25} = C_2$

$C_2 = \frac{28}{25}$

$f(x) = -\cos x + \frac{e^{5x}}{25} + \frac{2x}{5} + \frac{28}{25}$