Integration Methods

1) Integration by power rule. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$\int \left(x^3 + 3x - 2\right) dx = \frac{x^4}{4} + \frac{3x^2}{2} - 2x + C$$

2) Integration using derivative rules:

$$\int \sin u\, du = -\cos u + C$$
$$\int \cos u\, du = \sin u + C$$
$$\int \sec^2 u\, du = \tan u + C$$
$$\int \sec u \tan u\, du = \sec u$$
$$\int \csc^2 u\, du = -\cot u + C$$
$$\int \csc u \cot u\, du = -\csc u + C$$
$$\int \tan u\, du = -\ln|\cos u| + C$$
$$\int \sec u\, du = \ln|\sec u + \tan u| + C$$
$$\int \csc u\, du = -\ln|\csc u + \cot u| + C$$
$$\int \cot u\, du = \ln|\sin x| + C$$
$$\int \frac{1}{u^2+1} du = \tan^{-1} u + C$$
$$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + C$$
$$\int \frac{1}{|u|\sqrt{u^2-1}} du = \sec^{-1} u + C$$
$$\int a^u du = \frac{a^u}{\ln a} + C$$
$$\int e^u du = e^u + C$$
$$\int \frac{1}{u} du = \ln|u| + C$$

3) Integration by *substitution* (i.e. *u*, *du*)

$$\int x\sqrt{x^2+3}\, dx \qquad \text{Let } u = x^2 + 3,$$
$$du = 2x\, dx$$

$$\int x\sqrt{x^2+3}\, dx = \int u^{1/2}\left(\frac{1}{2}\right) du = \frac{1}{2}\int u^{1/2} dw = \frac{1}{2}\left(\frac{2}{3}\right)u^{3/2} = \frac{1}{3}(x^2+3)^{3/2} + C$$

*Sometimes you will to use advanced u-sub if the x's don't cancel. This will work most often when u is linear.

4) Integration by *trig identities*

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx$$

Now, let $u = \cos x$, $du = -\sin x$

$$\int \frac{\sin x}{\cos x} dx = -\int \frac{1}{u} du = -\ln|\cos x| + C$$

5) Integration by *Factoring*

$$\int \frac{x+1}{x^2+4x+3} dx$$

$$\int \frac{x+1}{x^2+4x+3} dx = \int \frac{x+1}{(x+1)(x+3)} dx = \int \frac{1}{(x+3)} dx = \ln|x+3| + C$$

6) Integration by *conjugate*

$$\int \frac{1}{1-\sin x} dx$$

$$\int \frac{1}{1-\sin x} dx = \int \frac{1}{1-\sin x}\frac{1+\sin x}{1+\sin x} dx = \int \frac{1+\sin x}{1-\sin^2 x} dx = \int \frac{1+\sin x}{\cos^2 x} dx$$

$$\int \frac{1}{\cos^2 x} dx + \int \frac{\sin x}{\cos^2 x} dx = \tan x + \frac{1}{\cos x} + C = \tan x + \sec x + C$$

7) Integration by *long division*

$$\int \frac{x^2+x+1}{x^2+1} dx = \int \left(1 + \frac{x}{x^2+1}\right) dx = x + \frac{1}{2}\ln(x^2+1) + C$$

8) Integration by *completing the square*

$$\int \frac{1}{x^2-4x+7} dx$$

$$\int \frac{1}{x^2-4x+7} dx = \int \frac{1}{(x^2-4x+4)+7-4} dx =$$

$$\int \frac{1}{(x-2)^2+3} dx = \int \frac{\frac{1}{3}}{\frac{(x-2)^2}{3}+1} dx =$$

$$\frac{1}{\sqrt{3}} \arctan \frac{x-2}{\sqrt{3}} + C$$