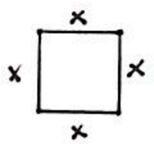
Related Rates Day 1 HWK
AB Calculus

name Key

1) Let A be the area of a square whose sides have length x, and assume x varies with time. At a certain instant the sides are 3 feet long and growing at a rate of 2 ft/min. How fast is the area growing at that instant?



when $x = 3$ ft

$\frac{dx}{dt} = 2$ ft/min

find $\frac{dA}{dt}$

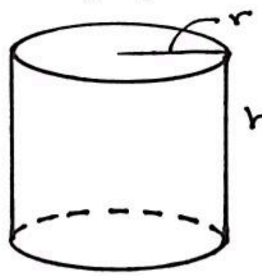
$A = x^2$

$\frac{dA}{dt} = 2x \frac{dx}{dt}$

$\left.\frac{dA}{dt}\right|_{x=3} = 2 \cdot 3 \cdot 2$

The area of the square is growing at a rate of 12 ft²/min when x = 3.

2) Let V be the volume of a cylinder having height h and radius r, and assume that h and r vary with time. At a certain instant, the height is 6 inches and increasing at 1 in/sec while the radius is 10 inches and decreasing at 1 in/sec. How fast is the volume changing at that instant? Is the volume increasing or decreasing?



$h = 6$; $\left.\frac{dh}{dt}\right|_{h=6} = 1$ in/sec

$r = 10$; $\left.\frac{dr}{dt}\right|_{r=10} = -1$ in/sec

$V = \pi r^2 \cdot h$

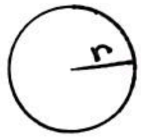
$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{h=6} = \pi(100)(1) + \pi \cdot 120 \cdot -1$

when $h = 6$

The volume is decreasing @ a rate of -20π in³/sec.

3) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How rapidly is the area enclosed by the ripple increasing at the end of 10 seconds?



$\frac{dr}{dt} = 3$ ft/sec
{constant}

r = 30 after 10 secs.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{t=10} = 2\pi \cdot 30(3)$

when t = 10, the area is increasing at a rate of 180π ft²/sec.

4) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 miles squared per hour. How fast is the radius of the spill increasing when the area is 9 miles squared?

$\frac{dA}{dt} = 6\ mi^2/hr$

{constant}

$\frac{dr}{dt} =$ ____ when $A=9$

$A = \pi r^2$

$9 = \pi r^2$

$r = \sqrt{\frac{9}{\pi}}$

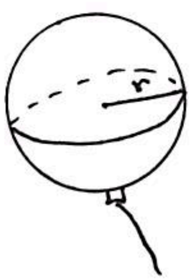
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$6 = 2\pi \sqrt{\frac{9}{\pi}} \cdot \frac{dr}{dt}\Big|_{A=9}$

$\frac{dr}{dt} = \frac{1}{\sqrt{\pi}}$ or $.564$

The radius is increasing at a rate of $\frac{1}{\sqrt{\pi}}$ mi/hr. when $A=9$.

5) A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?



$\frac{dr}{dt} = -15\ cm/min$

{constant}

find $\frac{dV}{dt}\Big|_{r=9}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt}\Big|_{r=9} = 4\pi(81)(-15)$

The air must be removed at a rate of -4860π cm³/min when $R=9$.