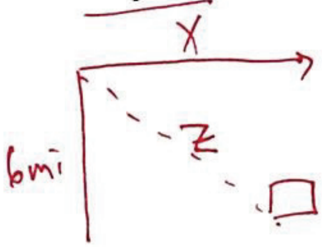
AP Calculus Name ________________

2.6 Related Rates Day 2

1) An airplane is flying at an altitude of 6 miles on a flight path that will take it directly over a radar tracking station. If the distance between the plane and the tracking station is decreasing at a rate of 400 miles per hour when the distance is 10 miles, what is the speed of the plane?



$\frac{dz}{dt} = 400$ mi/hr

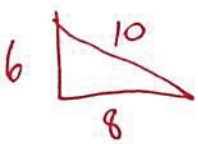
when $z = 10$

$x = 8$

Find $\frac{dx}{dt}$

$6^2 + x^2 = z^2$

$0 + 2x\frac{dx}{dt} = 2z\frac{dz}{dt}$



The speed of the plane is 500 mph.

$2(8)\frac{dx}{dt} = 2(10)(400)$

$\frac{dx}{dt} = \frac{2(10)(400)}{2(8)} = 500$ mi/hr

2) A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/sec, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

Find $\frac{dy}{dt}$

when $y = 8$

$x^2 + y^2 = 17^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = 5$ ft/sec

$x = 15$

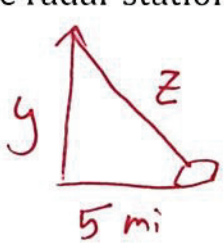
$2(15)(5) + 2(8)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{2(15)(5)}{16}$

$\frac{dy}{dt} = -9.375$ ft/sec

The ladder is moving down at a rate of 9.375 ft/sec.

3) A rocket, rising vertically, is tracked by a radar station that is on the ground 5 mi from the launch pad. How fast is the rocket rising when it is 4 miles high and its distance from the radar station is increasing at a rate of 2000 mi/hr?



Find $\frac{dy}{dt}$

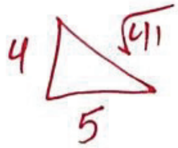
when $y = 4$

and $\frac{dz}{dt} = 2000$ mi/hr

$5^2 + y^2 = z^2$

$0 + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dy}{dt} = \frac{2(\sqrt{41})(2000)}{2(4)}$



$\frac{dy}{dt} = 500\sqrt{41}$

≈ 3201.562 mi/hr

The rocket is rising at a rate of 3201.562 mi/hr

4) A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 cubic feet per minute, how fast is the depth of the water increasing when the water is 16 ft deep?

$\frac{r}{h} = \frac{10}{24} = \frac{5}{12}$

so $r = \frac{5}{12}h$

$\frac{dV}{dt} = 20 \text{ ft}^3/\text{min}$

$\boxed{\text{Find } \frac{dh}{dt}}$ when $h = 16$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{5}{12}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{25}{144}h^3$

$V = \frac{25\pi}{432}h^3$

$\frac{dV}{dt} = \frac{75}{432}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{25}{144}\pi h^2 \frac{dh}{dt}$

$20 = \frac{25}{144}\pi (16)^2 \frac{dh}{dt}$

$20 = \frac{6400\pi}{144}\frac{dh}{dt}$

$\frac{20 \cdot 144}{6400\pi} = \frac{dh}{dt}$

$\approx .143$ ft/min

5) Sand pouring from a chute forms a conical pile whose height and diameter are always the same. If the height increases at a constant rate of 5 ft/min, at what rate is sand pouring from the chute when the pile is 10 ft high?

$d = h$ so $r = \frac{1}{2}h$

$\frac{dh}{dt} = 5$ ft/min

Find $\frac{dV}{dt}$ when $h = 10$

Note: when $h = 10$, $r = 5$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{1}{4}h^3$

$V = \frac{\pi}{12}h^3$

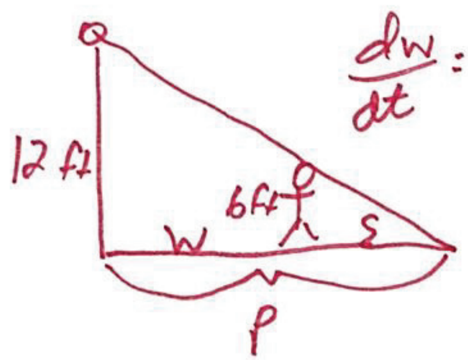
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi (10)^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 25\pi \cdot 5$

$\frac{dV}{dt} = 125\pi \text{ ft}^3/\text{min}$

6) A street light is at the top of a 12 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 30 ft from the base of the pole?



$\frac{dw}{dt} = 5$ ft/sec

Find $\frac{dp}{dt}$ when $w = 30$

$\frac{6}{12} = \frac{p - w}{p}$

$6p = 12p - 12w$

$12w = 6p$

$2w = p$

$2\frac{dw}{dt} = \frac{dp}{dt}$

$2(5) = \frac{dp}{dt}$

$\boxed{10 \text{ ft/sec}}$