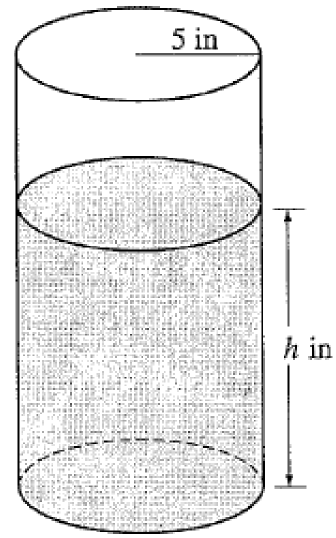
AP Differential Free Response Packet name ______________________

1) A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure. Let $h$ be the depth of the coffee in the pot, measured in inches, where $h$ is a function of time $t$, measured in seconds. The volume $V$ of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume $V$ of a cylinder with radius $r$ and height $h$ is $V = \pi r^2 h$.)




(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for $h$ as a function of $t$.

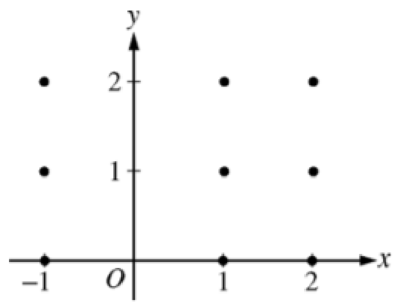
(c) At what time $t$ is the coffeepot empty?

## 2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   **(Note: Use the axes provided in the exam booklet.)**




   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

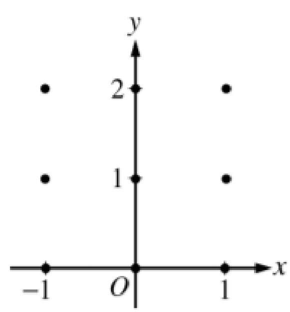
   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

## 2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   **(Note: Use the axes provided in the exam booklet.)**




   (b) Find $\frac{d^2y}{dx^2}$ in terms of $x$ and $y$. Describe the region in the $xy$-plane in which all solution curves to the differential equation are concave up.

   (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does $f$ have a relative minimum, a relative maximum, or neither at $x = 0$ ? Justify your answer.

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function $W$ models the total amount of solid waste stored at the landfill. Planners estimate that $W$ will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. $W$ is measured in tons, and $t$ is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of $W$ at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of $W$. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.