**Notes – Differential Equations** name Key
**Section 6.3**

1) $\frac{dy}{dx} = 5x^4$ $\quad$ $x \; y$ (0, 2)

$\int dy = \int 5x^4 dx$

$y = x^5 + C$

$2 = 0^5 + C$

$C = 2$

$y = x^5 + 2$

The point must work in the equation!

2) $\frac{dy}{dx} = \frac{2x}{y}$ $\quad$ y(1) = -3

$\int y\, dy = \int 2x\, dx$

$\frac{y^2}{2} = x^2 + C$

$\frac{9}{2} = 1^2 + C$

$C = \frac{7}{2}$

$2 \cdot \left( \frac{y^2}{2} = x^2 + \frac{7}{2} \right)$

$\sqrt{y^2} = \pm\sqrt{2x^2 + 7}$

since y(1) = -3 we have to pick $-\sqrt{\phantom{x}}$.

$y = -\sqrt{2x^2 + 7}$

3) $y' = 9x^2 y$ $\quad$ y(0) = 2

$\frac{dy}{dx} = 9x^2 y$

$\int \frac{dy}{y} = \int 9x^2\, dx$

$\ln|y| = 3x^3 + C$

$\ln 2 = 3(0)^3 + C$

$C = \ln 2$

$\ln|y| = 3x^3 + \ln 2$

$y = e^{3x^3 + \ln 2}$

$y = e^{3x^3} \cdot e^{\ln 2}$

$y = 2e^{3x^3}$

4) $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ $\quad$ y(0) = 0.5

$\int e^{2y} dy = \int 3x^2 dx$

$u = 2y$

$\frac{du}{dy} = 2$

$dy = \frac{du}{2}$

$\frac{e^{2y}}{2} = x^3 + C$

$\frac{e^1}{2} = 0 + C$

$C = \frac{e}{2}$

$\left( \frac{e^{2y}}{2} = x^3 + \frac{e}{2} \right) \cdot 2$

$e^{2y} = 2x^3 + e$

$2y = \ln(2x^3 + e)$

$y = \frac{1}{2} \ln(2x^3 + e)$

## Verifying Solutions to Differential Equations

5) Determine whether the function is a solution of the differential equation $y'' - y = 0$.

a) $y = \sin(x)$

$y' = \cos x \qquad y'' = -\sin x$

$-\sin x - \sin x = 0$ ?

No

b) $y = 4e^{-x}$

$y' = -4e^{-x} \qquad y'' = 4e^{-x}$

$4e^{-x} - 4e^{-x} = 0$

Yes!

$y = 4e^{-x}$ is a solution of $y'' - y = 0$

6) Let f be a function with f(1) = 4 such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where x = 1.

$slope = \frac{3(1)^2+1}{2(4)} = \frac{1}{2}$ when $x=1$

b) Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2).

$y-4 = \frac{1}{2}(x-1)$

$y-4 = \frac{1}{2}(1.2-1)$

$y = 4.1$

$f(1.2) \approx 4.1$

c) Find f(x) by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition f(1) = 4.

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$16 = 1 + 1 + C$

$C = 14$

$y^2 = x^3 + x + 14$

$y = \pm\sqrt{x^3+x+14}$

$f(x) = \pm\sqrt{x^3+x+14}$

7) Exponential Growth and Decay

The rate of growth is directly proportional to the population.

$\frac{dP}{dt} = k \cdot P$

$\int \frac{dP}{P} = \int k\,dt$

$\ln|P| = kt + C$

$e^{kt+C} = P$

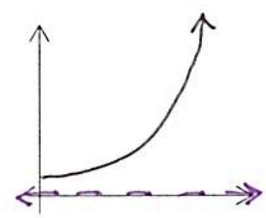
$P = e^{kt} \cdot \cancel{e^{C}}$
C

$P = Ce^{kt}$

growth constant → $k$

time → $t$

Initial Population → $C$



Example: A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If **the weight** of the puppy during its first 6 months **is increasing at a rate proportional to its weight**, then how much will the puppy weigh when it is 3 months old?

| t | w |
|---|---|
| 0 | 2.0 |
| 2 | 3.5 |
| 3 | ? |

$\frac{dw}{dt} = kt$

$w = Ce^{kt}$

$w = 2e^{kt}$

$3.5 = 2e^{k \cdot 2}$

$(1.75)^{\frac{1}{2}} = (e^{2k})^{\frac{1}{2}}$

$e^{k} = 1.75^{\frac{1}{2}}$

$w = 2(1.75)^{\frac{1}{2}t}$

$w(3) = 2(1.75)^{3/2} = 4.630$ lbs