Worksheet – Differential Equations **name** ____________________

Section 6.3

nd the particular equation for each differential equation and initial condition.

1) $\frac{dy}{dx} = y^2$ y(1) = -2

2) $\frac{dy}{dx} = \frac{x^3}{y^2}$ y(1) = 2

3) $\frac{dy}{dx} = \frac{1}{x}$ (1, 2)

4) $\frac{dy}{dx} = \frac{1}{3y^2}$ (2, -2)

5) $\frac{dy}{dx} = \frac{2x}{y}$ (2, 4)

6) $\frac{dy}{dx} = \frac{1+x}{xy}$ y(1) = -4

7) $\frac{dy}{dx} = \frac{e^x}{y}$ y(0) = 4

8) $xy\frac{dy}{dx} - \ln x = 0$ y(1) = 0

Verify whether or not the following are solutions to the given differential equations:

Differential	Potential Solution
$y'' + y = 0$	$y = C_1 \sin x - C_2 \cos x$
$y'' + 4y' = 2e^x$	$y = \frac{2}{5}(e^{-4x} + e^x)$

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10) Given $\frac{dy}{dx} = \frac{-4x+2}{y}$.

a) Write the equation for the line tangent to the graph at (2, -4) and use it to approximate f(1.8).

b) Find the particular solution y = f(x) to the differential equation with initial condition f(2) = -4.

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11) Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. Find the population in 2013 if the population was 12,000 in 1980 and the population doubles every 10 years.

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12) The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1599 years. What percent of a present amount will remain after 25 years?

11) $\frac{dy}{dt} = ky \quad \therefore \quad y = Ce^{kt}$

t	y
1980 → 0	12,000
1990 → 10	24,000
2013 → 33	?

$y = 12{,}000\,e^{kt}$

$24{,}000 = 12{,}000\,e^{k \cdot 10}$

$2 = e^{10k}$

$e^{k} = 2^{\frac{1}{10}}$

$y = 12{,}000\,(2)^{\frac{1}{10}t}$

$y(33) = 12{,}000 \cdot 2^{33/10} = 118{,}189$ people

12. $\frac{dR}{dt} = kR \quad \therefore \quad R = Ce^{kt}$

t	%
0	100
1599	50
25	

$R = 100e^{kt}$

$50 = 100\,e^{k \cdot 1599}$

$\frac{1}{2} = e^{1599k}$

$e^{k} = \left(\frac{1}{2}\right)^{\frac{1}{1599}}$

$R = 100\,(.5)^{\frac{1}{1599}t}$

$R(25) = 100\,(.5)^{\frac{25}{1599}}$

$= 98.922\,\%$

1) $\int \frac{dy}{y^2} = \int dx \qquad y(1) = -2$

$\int y^{-2}\,dy = \int dx$

$-\frac{1}{y} = x + C$

$\frac{1}{2} = 1 + C$

$C = -\frac{1}{2}$

$-\frac{1}{y} = x - \frac{1}{2}$

$\frac{-1}{y} = \frac{2x-1}{2}$

$-y = \frac{2}{2x-1}$

$y = \frac{-2}{2x-1}$

2) $\frac{dy}{dx} = \frac{x^3}{y^2} \qquad y(1) = 2$

$\int y^2\,dy = \int x^3\,dx$

$\frac{y^3}{3} = \frac{x^4}{4} + C$

$\frac{8}{3} = \frac{1}{4} + C$

$C = -\frac{4}{3}$

$\frac{y^3}{3} = \frac{x^4}{4} - \frac{4}{3}$

$y^3 = \frac{3x^4}{4} - 4$

$y = \sqrt[3]{\frac{3x^4}{4} - 4}$

3) $\int dy = \int \frac{1}{x}\,dx \qquad (1,2)$

$y = \ln|x| + C$

$2 = \ln 1 + C$

$C = 2$

$y = \ln|x| + 2$

4) $\int 3y^2\,dy = \int dx \qquad (2,-2)$

$y^3 = x + C$

$-8 = 2 + C$

$C = -10$

$y^3 = x - 10$

$y = \sqrt[3]{x - 10}$

5) $\int y\,dy = \int 2x\,dx \quad (2,4)$

$\frac{y^2}{2} = x^2 + C$

$8 = 4 + C$

$C = 4$

$\frac{y^2}{2} = x^2 + 4$

$y^2 = 2x^2 + 8$

$y = \sqrt{2x^2+8}$

6) $\int y\,dy = \int \frac{1+x}{x}\,dx \quad y(1) = -4$

$\frac{y^2}{2} = \int \frac{1}{x}dx + \int 1\,dx$

$\frac{y^2}{2} = \ln|x| + x + C$

$8 = \ln 1 + 1 + C$

$7 = C$

$\frac{y^2}{2} = \ln|x| + x + 7$

$y^2 = 2\ln|x| + 2x + 14$

$y = -\sqrt{2\ln|x| + 2x + 14}$

7) $\int y\,dy = \int e^x\,dx \quad y(0) = 4$

$\frac{y^2}{2} = e^x + C$

$8 = e^0 + C$

$C = 7$

$\frac{y^2}{2} = e^x + 7$

$y^2 = 2e^x + 14$

$y = \sqrt{2e^x + 14}$

8) $xy\frac{dy}{dx} = \ln x$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$\int y\,dy = \int \frac{\ln x}{x}\,dx$

$y(1) = 0$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$

$0 = \frac{(\ln 1)^2}{2} + C$

$C = 0$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2}$

$y^2 = (\ln x)^2$

$y = \ln x$

9) a) $y'' + y = 0$

$y' = C_1 \cos x + C_2 \sin x$

$y'' = -C_1 \sin x + C_2 \cos x$

$\cancel{-C_1 \sin x} + \cancel{C_2 \cos x} + \cancel{C_1 \sin x} - \cancel{C_2 \cos x} = 0\ ?$

yes!

b) $y'' + 4y' = 2e^x$

$y' = \frac{2}{5}(e^{-4x} \cdot -4 + e^x)$

$y'' = \frac{2}{5}(16e^{-4x} + e^x)$

$\frac{2}{5}(16e^{-4x} + e^x) + 4\left[\frac{2}{5}(-4e^{-4x} + e^x)\right] = 2e^x\ ?$

$\cancel{\frac{32}{5}e^{-4x}} + \frac{2}{5}e^x + \cancel{\frac{-32}{5}e^{-4x}} + \frac{8}{5}e^x = 2e^x\ ?$

$\frac{10}{5}e^x = 2e^x\ ?$

yes!

10) $\frac{dy}{dx} = \frac{-4x+2}{y}$

$m @ (2,-4) = \frac{-6}{-4} = \frac{3}{2}$

a) $y + 4 = \frac{3}{2}(x-2)$

$y + 4 = \frac{3}{2}(1.8 - 2)$

$y + 4 = \frac{3}{\cancel{2}}\left(\frac{-\cancel{2}}{10}\right)$

$f(1.8) \approx 3\ 7/10$

$y = -3\frac{7}{10}$

b) $\int y\, dy = \int (-4x+2)\, dx$

$(2,-4)$

$\frac{y^2}{2} = -2x^2 + 2x + C$

$\frac{y^2}{2} = -2x^2 + 2x + 12$

$8 = -8 + 4 + C$

$C = 12$

$y^2 = -4x^2 + 4x + 24$

$y = -\sqrt{-4x^2 + 4x + 24}$