Derivative Practice Worksheet

Find the derivative of the given function with respect to x.
Show all necessary work.

1. $y = 3$

$\boxed{y' = 0}$

2. $f(x) = -2$

$\boxed{f'(x) = 0}$

3. $y = 5 + \sin x$

$y' = 0 + \cos x$

$\boxed{y' = \cos x}$

4. $g(x) = x^2 + 4$

$\boxed{g'(x) = 2x}$

5. $s(t) = t^3 - 2t + 4$

$\boxed{s'(t) = 3t^2 - 2}$

6. $y = \sin x$

$\boxed{y' = \cos x}$

7. $y = \tan x$

$\boxed{y' = \sec^2 x}$

8. $y = \cot x$

$\boxed{y' = -\csc^2 x}$

9. $y = \cos x$

$\boxed{y' = -\sin x}$

10. $y = \csc x$

$\boxed{y' = -\csc x \cot x}$

11. $f(x) = x^2 - \frac{1}{2}\cos x$

$f'(x) = 2x - \frac{1}{2}(-\sin x)$

$\boxed{f'(x) = 2x + \frac{1}{2}\sin x}$

12. $g(x) = \frac{1}{x} - 3\sin x$ x^{-1}

$\boxed{g'(x) = \frac{-1}{x^2} - 3\cos x}$

13. $f(x) = \sec x + 3x$

$\boxed{f'(x) = \sec x \tan x + 3}$

14. $g(t) = \pi \cos t$

$\boxed{g'(t) = -\pi \sin t}$

15. $h(x) = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$

$\boxed{h'(x) = \frac{-1}{x^4}}$

16. $y = \frac{\sqrt{x}}{x} = \frac{x^{\frac{1}{2}}}{x^1} = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$\boxed{y' = \frac{-1}{2\sqrt{x^3}}}$

17. $f(x) = x^3 - 3x - 2x^{-4}$

$\boxed{f'(x) = 3x^2 - 3 + 8x^{-5}}$

18. $y = \frac{3x-2}{2x-3}$

$y' = \frac{(2x-3)(3) - (3x-2)(2)}{(2x-3)^2}$

$y' = \frac{6x - 9 - 6x + 4}{(2x-3)^2}$

$\boxed{y' = \frac{-5}{(2x-3)^2}}$

19. $g(x) = (x^2 - 2x + 1)(x^3 - 1)$

$g'(x) = (x^3-1)(2x-2) + (x^2-2x+1)(3x^2)$

$g'(x) = 2x^4 - 2x^3 - 2x + 2 + 3x^4 - 6x^3 + 3x^2$

$\boxed{g'(x) = 5x^4 - 8x^3 + 3x^2 - 2x + 2}$

20. $y = xcosx$

$y' = \cos x(1) + x(-\sin x)$

$\boxed{y' = \cos x - x\sin x}$

21. $f(x) = \frac{x+1}{\sqrt{x}}$

$f'(x) = \dfrac{\sqrt{x}(1) - (x+1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$

$= \dfrac{\frac{\sqrt{x}}{1} \cdot \frac{2\sqrt{x}}{2\sqrt{x}} - \frac{x+1}{2\sqrt{x}}}{x} = \dfrac{2x - x - 1}{2\sqrt{x}} \cdot \dfrac{1}{x}$

$= \boxed{\dfrac{x-1}{2x\sqrt{x}}}$

22. $f(x) = (x + 1)cosx$

$f'(x) = \cos x(1) + (x+1)(-\sin x)$

$\boxed{f' = \cos x - x\sin x - \sin x}$

23. $y = x + cotx$

$\boxed{y' = 1 - \csc^2 x}$

24. $g(x) = \sqrt{x} + 4\sec x$

$\boxed{g'(x) = \dfrac{1}{2\sqrt{x}} + 4\sec x\tan x}$

25. $y = -\csc x - \sin x$

$\boxed{y' = \csc x\cot x - \cos x}$

26. $f(x) = x^2 \tan x$

$f'(x) = \tan x(2x) + (x^2)(\sec^2 x)$

$\boxed{f'(x) = 2x\tan x + x^2\sec^2 x}$

27. $f(x) = (3x - 2x^2)^3$

S: $(\quad)^3$ → $3(3x-2x^2)^2$

C: $3x - 2x^2$ → $3 - 4x$

$\boxed{f'(x) = (9 - 12x)(3x-2x^2)^2}$

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28. $g(x) = \frac{e^x}{1-e^x}$

$$g'(x) = \frac{(1-e^x)(e^x) - (e^x)(-e^x)}{(1-e^x)^2}$$

$$g'(x) = \frac{e^x - e^{2x} + e^{2x}}{(1-e^x)^2}$$

$$\boxed{g'(x) = \frac{e^x}{(1-e^x)^2}}$$

29. $y = \left(\frac{3x-1}{x^2+3}\right)^2$

$$\ln y = 2\ln(3x-1) - 2\ln(x^2+3)$$

$$\cancel{y} \cdot \frac{1}{\cancel{y}}\left(\frac{dy}{dx}\right) = \left[\frac{6}{3x-1} - \frac{4x}{x^2+3}\right] \cdot y$$

$$\boxed{\frac{dy}{dx} = \left[\frac{6}{3x-1} - \frac{4x}{x^2+3}\right]\left(\frac{3x-1}{x^2+3}\right)^2}$$

30. $y = \cos(3x)^2$

$$\left.\begin{array}{l} S: \cos(\ \ ) \\ C: (3x)^2 \\ P: 3x \end{array}\right\} \begin{array}{l} -\sin((3x)^2) \\ 2(3x) \\ 3 \end{array}$$

$$\boxed{y' = -18x \cdot \sin(3x)^2}$$

31. $y = cos^2(3x) = [\cos(3x)]^2$

$$\left.\begin{array}{l} S: [\ \ ]^2 \\ C: \cos(3x) \\ P: 3x \end{array}\right\} \begin{array}{l} 2[\cos(3x)] \\ -\sin(3x) \\ 3 \end{array}$$

$$\boxed{y' = -6\sin(3x)\cos(3x)}$$

32. $f(x) = sin^3 4x = [\sin 4x]^3$

$$\left.\begin{array}{l} S: [\ \ ]^3 \\ C: \sin(4x) \\ P: 4x \end{array}\right\} \begin{array}{l} 3[\sin(4x)]^2 \\ \cos(4x) \\ 4 \end{array}$$

$$\boxed{f'(x) = 12\sin^2(4x)\cos(4x)}$$

33. $y = ln\sqrt{\frac{x+3}{x-2}} = \frac{1}{2}\ln(x+3) - \frac{1}{2}\ln(x-2)$

$$y' = \frac{1}{2(x+3)} - \frac{1}{2(x-2)}$$

$$\boxed{y' = \frac{1}{2x+6} - \frac{1}{2x-4}}$$

34. $y^3 = x$

$$\frac{3y^2\left(\frac{dy}{dx}\right)}{3y^2} = \frac{1}{3y^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{3y^2}}$$

35. $y = x2^x$

$$y' = 2^x(1) + x(2^x \cdot \ln 2)$$

$$\boxed{y' = 2^x(1 + x\ln 2)}$$

36. $y = \frac{1}{x-2} = (x-2)^{-1}$

S: $(\quad)^{-1}$ $-1(x-2)^{-2}$

C: $x-2$ $\quad 1$

$$\boxed{y' = \frac{-1}{(x-2)^2}}$$

37. $y = \cos 3x^2$

S: $\cos(\quad)$ $-\sin(3x^2)$

C: $3x^2$ $\quad 6x$

$$\boxed{y' = -6x\sin 3x^2}$$

38. $y = \sqrt[3]{9x^2 + 4}$

S: $(\quad)^{1/3}$ $\frac{1}{3}(9x^2+4)^{-2/3}$

C: $9x^2+4$ $\quad 18x$

$$\boxed{y' = \frac{6x}{\sqrt[3]{(9x^2+4)^2}}}$$

39. $y = (\cos 3)x^2$

$$\boxed{y' = (2\cos 3)x}$$

*cos3 is a constant

40. $y = \frac{x}{2^{3x}}$

$$y' = \frac{2^{3x}(1) - x(2^{3x} \cdot \ln 2 \cdot 3)}{(2^{3x})^2}$$

$$y' = \frac{\cancel{2^{3x}}(1 - 3\ln 2 x)}{(\cancel{2^{3x}})^2} = \frac{1 - x\ln 2^3}{2^{3x}}$$

$$\boxed{y' = \frac{1 - x\ln 8}{2^{3x}}}$$

41. $x^3 - xy + y^2 = 4$

$$3x^2 - y(1) - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{\frac{dy}{dx}(-x+2y)}{-x+2y} = \frac{-3x^2+y}{-x+2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2+y}{-x+2y}}$$

42. $y = (x+1)^x$

Function to a functional power so you have to use log diff

$\ln y = x \cdot \ln(x+1)$

$\uparrow$ product rule

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \ln(x+1)(1) + (x)\left(\frac{1}{x+1}\right)$$

$$\cancel{y} \cdot \cancel{\frac{1}{y}}\left(\frac{dy}{dx}\right) = \left[\ln(x+1) + \frac{x}{x+1}\right] \cdot y$$

$$\boxed{\frac{dy}{dx} = \left[\ln(x+1) + \frac{x}{x+1}\right](x+1)^x}$$

$y = 5\log_3(x^2+1)^2$ $= 10\log_3(x^2+1)$

$y' = \dfrac{10(2x)}{(x^2+1)\ln 3}$

$\boxed{y' = \dfrac{20x}{(x^2+1)\ln 3}}$

44. Find f' if $f(\theta) = \theta\cos\theta$ *Product Rule

$f'(\theta) = \cos\theta(1) + (\theta)(-\sin\theta)$

$\boxed{f'(\theta) = \cos\theta - \theta\sin\theta}$

45. Find $\frac{dy}{dx}$ if $y = \frac{x^2+5x-3}{2x+7}$

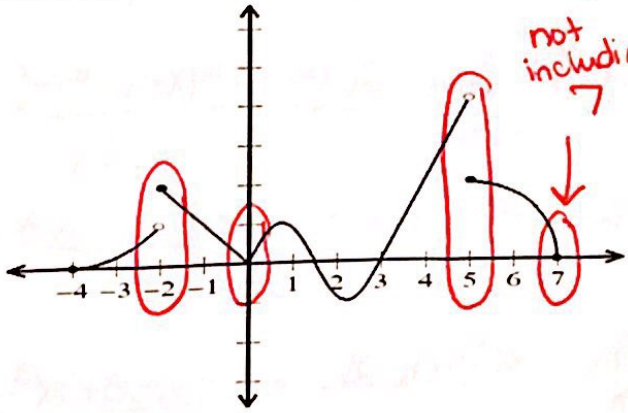
$\frac{dy}{dx} = \dfrac{(2x+7)(2x+5)-(x^2+5x-3)(2)}{(2x+7)^2}$

$\frac{dy}{dx} = \dfrac{4x^2+10x+14x+35-2x^2-10x+6}{(2x+7)^2}$

$\boxed{\frac{dy}{dx} = \dfrac{2x^2+14x+41}{(2x+7)^2}}$

46. Identify the x-values in the open interval (-4, 7) where the function is not differentiable.

not including 7



$\boxed{x = -2, 0, 5}$

47. If $f(x) = x^2 - 3x + 4$:

a. Find AROC over [-1,3]

$\dfrac{f(3)-f(-1)}{3-(-1)} = \dfrac{4-8}{4} = \dfrac{-4}{4} = \boxed{-1}$

b. Find the equation of the line tangent to $f(x)$ at x = 0.

$f'(x) = 2x-3$

$f'(0) = 2(0)-3 = \boxed{-3}$

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48. Use Limit Definition of a Derivative to find f'(x):
$f(x) = -2x^2 + 4x - 5$

$$f'(x) = \lim_{h \to 0} \frac{-2(x+h)^2 + 4(x+h) - 5 - (-2x^2 + 4x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 4x + 4h - 5 + 2x^2 - 4x + 5}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 \cancel{+4x} + 4h \cancel{-5} \cancel{+2x^2} \cancel{-4x} \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{-4xh - 2h^2 + 4h}{h} = \lim_{h \to 0} \frac{\cancel{h}(-4x - 2h + 4)}{\cancel{h}}$$

$$= \lim_{h \to 0} -4x - 2h + 4 = \boxed{-4x + 4}$$

49. Find $G'(x)$ if $G(x) = 6\sqrt[3]{5 - 4x}$ $= 6(5-4x)^{\frac{1}{3}}$

S: $6(\quad)^{1/3}$ — $2(5-4x)^{-2/3}$

C: $5 - 4x$ — -4

$$\boxed{G'(x) = \frac{-8}{\sqrt[3]{(5-4x)^2}}}$$

50. Find y' if
$y = (4x + 1)(1 - x)^3$

S: $(\quad)^3$ — $3(1-x)^2$

C: $1-x$ — -1

$$y' = (1-x)^3(4) + (4x+1)(-3(1-x)^2)$$

$$\boxed{y' = 4(1-x)^3 - 3(4x+1)(1-x)^2}$$

or

$$y' = (1-x)^2(4(1-x) - 3(4x+1))$$

$4 - 4x - 12x - 3$

$$\boxed{y' = (1-x^2)(-16x+1)}$$

51. Find f'(x) if $f(x) = \sin^2(3x + 2)$ $= [\sin(3x+2)]^2$

S: $[\quad]^2$ — $2[\sin(3x+2)]^1$

C: $\sin(3x+2)$ — $\cos(3x+2)$

P: $3x+2$ — 3

$$\boxed{f'(x) = 6\cos(3x+2)\sin(3x+2)}$$