Derivative Retest Ticket	Name: Key

Multiple Choice: Write the letter of the appropriate response to the left of the exercise number. Show all work.

E 1. If $f(2)=3$, $f(3)=2$, and $f'(2)=-2$, then find the equation of the normal line when $x=2$.

A. $y-2=-2(x-3)$

B. $y-2=-2(x-3)$

C. $y+3=2(x+2)$

D. $y-3=-\frac{1}{2}(x-2)$

E. None of these

normal → perpendicular

$m=\frac{1}{2}$

$y-3=\frac{1}{2}(x-2)$

B 2. If $f(x)=-x^2+x$, which if the following will calculate the derivative of $f(x)$?

A. $\lim\limits_{h\to 0}\dfrac{(-x^2+x+h)-(-x^2+x)}{h}$

B. $\lim\limits_{h\to 0}\dfrac{\left[\left(-(x+h)^2+(x+h)\right)\right]-(-x^2+x)}{h}$

C. $\dfrac{\left[\left(-(x+h)^2+(x+h)\right)\right]-(-x^2+x)}{h}$

D. $\dfrac{(-x^2+x+h)-(-x^2+x)}{h}$

E. None of these

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

B 3. Let

$f(3)=0$	$f'(3)=6$
$g(3)=1$	$g'(3)=\frac{1}{3}$

. Find $h'(3)$ *if* $h(x)=\dfrac{f(x)}{g(x)}$.

A. 18

B. 6

C. −6

D. −2

E. None of these

$h'=\dfrac{g\cdot f'-f\cdot g'}{[g(x)]^2}$

$h'(3)=\dfrac{1\cdot 6-0\cdot\frac{1}{3}}{1^2}$

C 4. Find the derivative of the function. $f(x) = -8x^2 - 4\cos x$

A. $f'(x) = -8x + 4\sin x$
B. $f'(x) = -16x - 4\sin x$
C. $f'(x) = -16x + 4\sin x$
D. $f'(x) = -16x - 4\cos x$
E. None of these

$f' = -16x + 4\sin x$

E 5. Find $\frac{d^2y}{dx^2}$ if $y = \frac{x+2}{x-3}$

A. $\frac{-2}{(x-3)^2}$
B. 0
C. $\frac{-10}{(x-3)^3}$
D. $\frac{2}{(x-3)^2}$
E. None of these

$y' = \frac{(x-3)\cdot 1 - (x+2)\cdot 1}{(x-3)^2}$

$= \frac{-5}{(x-3)^2} = -5(x-3)^{-2}$

s: $-5(\quad)^{-2}$

c: $(x-3)$

$y'' = 10(x-3)^{-3}$

E 6. Determine the value(s), if any, at which the graph of the function has a horizontal tangent.

$y = \frac{8}{x-6}$

$y' = \frac{-8}{(x-6)^2}$

$y = 8(x-6)^{-1}$

$0 = \frac{-8}{(x-6)^2}$

$0 \neq -8$

A. 8
B. 8 *and* 6
C. 8 *and* −6
D. 6
E. There are no values for which the function has a horizontal tangent line.

B 7. Find $r'(t)$ if $r(t) = (t^5 + 3)^4$

$r' = 4(t^5 + 3)^3 \cdot 5t^4$

A. $r'(t) = 4t^4(t^5 + 3)^3$
B. $r'(t) = 20t^4(t^5 + 3)^3$
C. $r'(t) = 20t^6(t^5 + 3)^3$
D. $r'(t) = 20t^5(t^5 + 3)^3$
E. None of these

A 8. Find the derivative if the function if $f(x)=x^5\sqrt{8-2x}$ $= x^5(8-2x)^{\frac{1}{2}}$

A. $f'(x)=\dfrac{x^4(40-11x)}{\sqrt{8-2x}}$

B. $f'(x)=\dfrac{x^4(40+11x)}{\sqrt{8-2x}}$

C. $f'(x)=\dfrac{x^4(4-11x)}{\sqrt{8-2x}}$

D. $f'(x)=\dfrac{x^4(40-x)}{\sqrt{8-2x}}$

E. None of these

$$f' = x^5 \cdot \tfrac{1}{2}(8-2x)^{-1/2} \cdot -2 + (8-2x)^{\frac{1}{2}} \cdot 5x^4$$

$$= \frac{-x^5}{\sqrt{8-2x}} + \frac{5x^4}{1} \cdot \frac{\sqrt{8-2x}}{1} \cdot \frac{\sqrt{8-2x}}{\sqrt{8-2x}}$$

$$= \frac{-x^5 + 5x^4(8-2x)}{\sqrt{8-2x}}$$

$$= \frac{-x^5+40x^4-10x^5}{\sqrt{8-2x}} = \frac{-11x^5+40x^4}{\sqrt{8-2x}} = \frac{x^4(-11x+40)}{\sqrt{8-2x}}$$

A 9. Find the derivative if the function if $f(x)=3\sec^2(5\pi x-3)$

A. $f'(x)=30\pi\sec^2(5\pi x-3)\tan(5\pi x-3)$

B. $f'(x)=30\sec^2(5\pi x-3)\tan(5\pi x-3)$

C. $f'(x)=5\pi\sec^2(5\pi x-3)\tan(5\pi x-3)$

D. $f'(x)=15\pi\sec^2(5\pi x-3)\tan(5\pi x-3)$

E. None of these

s: $3[\quad]^2$ | $6[\sec(5\pi x-3)]$

c: $\sec(\quad)$ | $\sec(5\pi x-3)\tan(5\pi x-3)$

P: $5\pi x-3$ | 5π

D 10. Find $\dfrac{dy}{dx}$ by implicit differentiation if $x^2+9x+9xy-y^2=16$

A. $\dfrac{dy}{dx}=\dfrac{x+9+9y}{y-9x}$

B. $\dfrac{dy}{dx}=\dfrac{2x+9+9y}{2x-9y}$

C. $\dfrac{dy}{dx}=\dfrac{2x-9+9y}{2y-9x}$

D. $\dfrac{dy}{dx}=\dfrac{2x+9+9y}{2y-9x}$

E. None of these

$$2x+9+9x\cdot\frac{dy}{dx}+9y-2y\frac{dy}{dx}=0$$

$$9x\frac{dy}{dx}-2y\frac{dy}{dx}=-2x-9-9y$$

$$\frac{dy}{dx}(9x-2y)=-2x-9-9y$$

$$\frac{dy}{dx}=\frac{-(2x+9+9y)}{9x-2y}$$

$$=\frac{2x+9+9y}{2y-9x}$$

C 11. Find $\frac{d^2y}{dx^2}$ in terms of x and y if $2-8xy=9x-5y$

A. $\frac{d^2y}{dx^2}=\frac{16(8y-9)}{(5-8x)^2}$

~~B.~~ $\frac{d^2y}{dx^2}=\frac{464}{(5-8x)^3}$

C. $\frac{d^2y}{dx^2}=\frac{16(8y+9)}{(5-8x)^2}$

~~D.~~ $\frac{d^2y}{dx^2}=\frac{16(8+9y)}{(5+8x)^2}$

E. None of these

$$-8x\frac{dy}{dx}-8y=9-5\frac{dy}{dx}$$

$$-8x\frac{dy}{dx}+5\frac{dy}{dx}=9+8y$$

$$\frac{dy}{dx}(-8x+5)=9+8y$$

$$\frac{dy}{dx}=\frac{9+8y}{-8x+5}$$

$$\frac{d^2y}{dx^2}=\frac{(-8x+5)\cdot 8\frac{dy}{dx}-(9+8y)\cdot -8}{(-8x+5)^2}$$

$$=\frac{(\cancel{-8x+5})8\cdot\frac{(9+8y)}{\cancel{-8x+5}}+72+64y}{(-8x+5)^2}$$

$$\frac{d^2y}{dx^2}=\frac{72+64y+72+64y}{(-8x+5)^2}$$

$$=\frac{144+128y}{(-8x+5)^2}$$

12. Suppose the functions f and g and their derivatives with respect to x have the following values at $x=0$ and $x=1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$\frac{1}{3}$
1	3	-4	$-\frac{1}{3}$	$-\frac{8}{3}$

Evaluate the derivative with respect to x of $f(g(x))$ at $x=0$. Show the work that leads to your solutions.

$$\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))\cdot g'(x)$$

$$f'(g(0))\cdot g'(0)=f'(1)\cdot g'(0)=-\frac{1}{3}\cdot\frac{1}{3}=-\frac{1}{9}$$

13. Let f and g be inverse functions.

The following table lists a few values of f, g, and f'.

x	$f(x)$	$g(x)$	$f'(x)$
2	1	9	$\frac{1}{3}$
9	2	10	$\frac{1}{12}$

$g'(2)=$

$g(2)=9$

$f(9)=2$

$$g'(2)=\frac{1}{f'(9)}=\frac{1}{\frac{1}{12}}$$

$$g'(2)=12$$

14-17. Find the derivative of each function. Show all work.

14. $\frac{d}{dx}(\mathrm{arcsec}(\ln x))$ $u = \ln x$

$du = \frac{1}{x}$

$$\frac{\frac{1}{x}}{|\ln x|\sqrt{(\ln x)^2 - 1}}$$

$$= \boxed{\frac{1}{x|\ln x|\sqrt{(\ln x)^2-1}}}$$

15. $y = \left(\frac{10x^3}{\sqrt{x+1}}\right)^4$

$\ln y = 4\ln 10x^3 - 4\ln(x+1)^{\frac{1}{2}}$

$\ln y = 4\ln 10 + 12\ln x - 2\ln(x+1)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{12}{x} - \frac{2}{x+1}$

$$\boxed{\frac{dy}{dx} = \left(\frac{12}{x} - \frac{2}{x+1}\right)\left(\frac{10x^3}{\sqrt{x+1}}\right)^4}$$

16. $y = (2x+1)6^x$

$y' = (2x+1)\cdot 6^x \cdot \ln 6 + 6^x(2)$

$$\boxed{y' = 6^x\left[(2x+1)\ln 6 + 2\right]}$$

17. $f(x) = (2x+1)^{3x}$

$y = (2x+1)^{3x}$

$\ln y = 3x\cdot\ln(2x+1)$

$\frac{1}{y}\cdot\frac{dy}{dx} = 3x\cdot\frac{2}{2x+1} + \ln(2x+1)\cdot 3$

$$\boxed{\frac{dy}{dx} = \left[\frac{6x}{2x+1} + 3\ln(2x+1)\right](2x+1)^{3x}}$$