Average Rate of Change vs Instantaneous Rate of Change Examples

(calc.)1. A roller coaster has its largest drop modeled by the equation $d(t) = 1.93t^3 - 31.82t^2 + 113.83t + 158.65$ where $d(t)$ is measured in feet and time t is measured in seconds.

a) What is the average speed of the roller coaster from 2 seconds to 8 seconds?

42.25 ft/sec

$\frac{d(8)-d(2)}{8-2} = \frac{20.97-274.47}{6} = -42.25$

b) What is the average speed of the roller coaster from 4 seconds to 6 seconds?

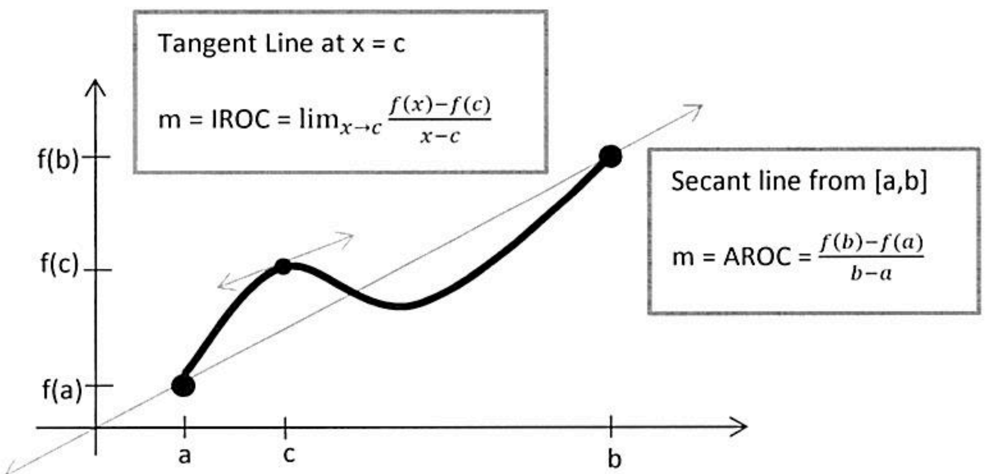
57.69 ft/sec

$\frac{d(6)-d(4)}{6-4} = \frac{112.99-228.37}{2} = -57.69$

c) Estimate the instantaneous speed of the roller coaster at exactly 5 seconds.

≈ 59.6 ft/sec

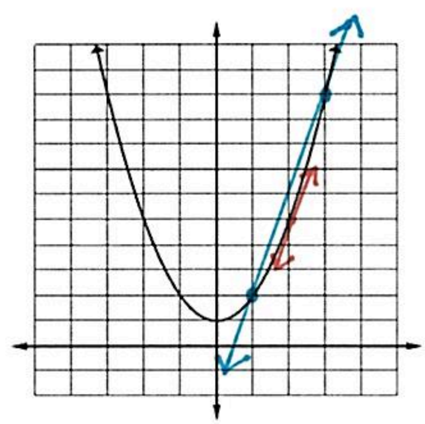
$\frac{d(5.1)-d(4.9)}{5.1-4.9} = \frac{167.561-179.481}{.2} = -59.6$



Example: $f(x) = x^2 + 1$

a) Find the average rate of change for f(x) from [1,3].

$\frac{f(3)-f(1)}{3-1} = \frac{10-2}{2} = 4$



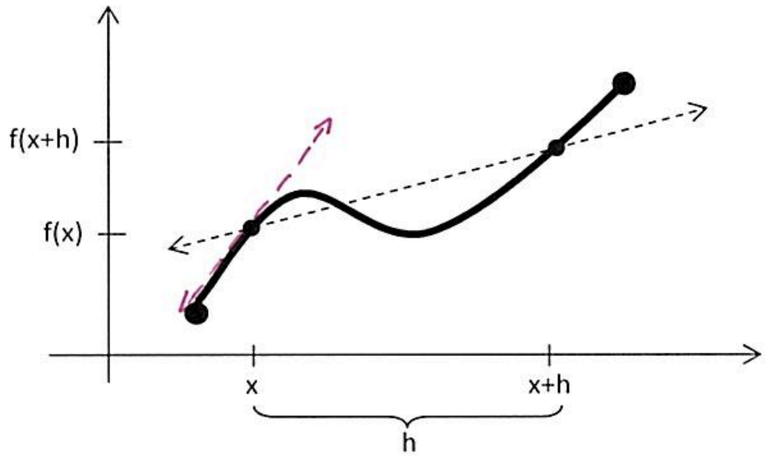
b) Find the instantaneous rate of f(x) at x = 2.

$\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2} \frac{x^2+1-5}{x-2}$

$= \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = 4$

What is a derivative?

A derivative is a formula for finding the slope of the tangent line(IROC) of a function at any x-value.



In order to find the derivative of a function at a point, the function must be continuous at that point, there can be no sharp turns, and you cannot be at an endpoint, or a vertical tangent.

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is the limit definition of a derivative. It is the basis for all of the derivative rules that we will learn this year.

Examples:

1) Find the derivative of $f(x) = x^2+1$ using the limit definition of a derivative.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = 2x$$

2) Find the derivative of $y = x^2 - 2x + 3$ using the limit definition of a derivative.

$$y' = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 3 - (x^2 - 2x + 3)}{h}$$

$$y' = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h}$$

$$y' = \lim_{h \to 0} \frac{h(2x+h-2)}{h} = 2x - 2$$