AB Retest Practice Name Key 2019

Applications of Differentiation 4.1-4.4, and f, f′, f″

1. What are all values of x for which the function $f(x) = x^3 - 6x^2 + 9x - 1$ is decreasing?

$f' = 3x^2 - 12x + 9$

$f' = 3(x^2 - 4x + 3)$

$0 = 3(x-3)(x-1)$

CV @ $x = 3, 1$

f': $+$ | 1 | $-$ | 3 | $+$

f is decreasing when $f' \le 0$

$\therefore$ on $[1, 3]$

2. Given the function $f(x) = x^4 + 2x^3$, find the relative extrema, and the points of inflection.

$f' = 4x^3 + 6x^2$

$f' = 2x^2(2x+3)$

$f'' = 12x(x+1)$

$0 = 12x(x+1)$

PPOI @ $x = 0, -1$

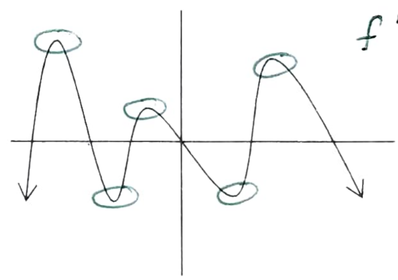
f'': $+$ | -1 | $-$ | 0 | $+$

f': $-$ | -1.5 | $+$ | 0 | $+$

Rel min @ $x = -1.5$

POI @ $x = -1, 0$

3. The figure shows the graph of the derivative of a function f. How many points of inflection does f have in the interval shown?



f has POI @ the relative extrema of f'

$\therefore$ there are 5 POI for $f(x)$.

4. Let f be the function given by $f(x) = x^3 - 3x$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval [-1,2]?

$f' = 3x^2 - 3$

AROC: $\dfrac{f(2) - f(-1)}{2 - (-1)} = \dfrac{2 - 2}{3} = 0$

$3x^2 - 3 = 0$

$x = \pm 1$

only $+1$ is on the given interval, $\therefore$ $c = 1$

5. What are all values of x for which the graph of $y = \frac{3}{3-x}$ is concave downward?

$y = 3(3-x)^{-1}$

$y' = -3(3-x)^{-2} \cdot -1 = 3(3-x)^{-2}$

$y'' = -6(3-x)^{-3} \cdot -1 = \dfrac{6}{(3-x)^3}$

$+$ | 3 | $-$

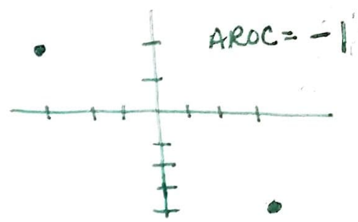
f is ccd on $(3, \infty)$

4. The function f is continuous for $-3 \le x \le 3$ and differentiable for $-3 < x < 3$. If $f(-3) = 2$ and $f(3) = -4$, which statement(s) must be true?

X I. There exists c, where $-3 < c < 3$, such that $f'(c) = 0$. x $f(a) \neq f(b)$

II. There exists c, where $-3 < c < 3$, such that $f'(c) = -1$. MVT

III. There exists c, where $-3 \le c \le 3$, such that $f(c) \ge f(x)$ for all x on the closed interval $-3 \le c \le 3$. EVT



* II & III

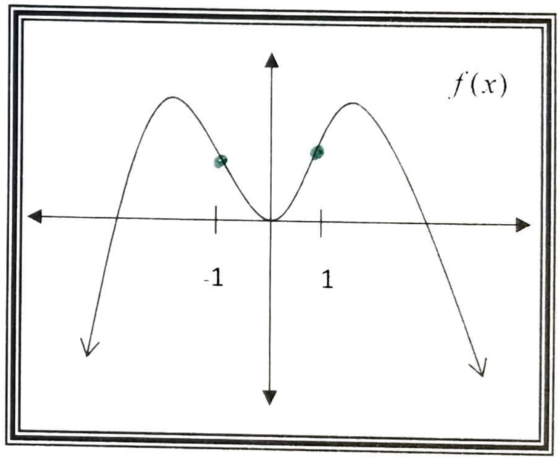
5. Let f''(x) = 4x³ – 2x and let f(x) have critical numbers -1, 0, and 1. Use the Second Derivative Test to determine if any of the critical numbers gives a relative minimum.

$f''(-1) = -4 + 2 < 0$ ∴ rel max @ $x = -1$

$f''(0) = 0$ ∴ inconclusive

$f''(1) = 4 - 2 > 0$ ∴ rel min @ $x = 1$

6. The graph of $f(x)$ is shown in the figure to the right. Where is f′ decreasing?



$f'' < 0$ ∴ CC↓

* $(-\infty, -1) \cup (1, \infty)$

7. Where does the absolute maximum value of the function $f(x) = x^3 - 3x + 1$ occur on the interval [-3,2]?

$f' = 3x^2 - 3$

$0 = 3(x-1)(x+1)$

CV @ $x = 1, -1$

x	f(x)
-3	-17
-1	3
1	-1
2	3

* Absolute max is 3 @ occurs @ $x = -1, 2$

8. Let f be a function that is continuous on the closed interval [0,3]. The function f and its derivatives have the properties indicated in the table below.

x	0	0<x<2	2	2<x<3	3	3<x<4	4
f(x)	1	+	0	+	+1	+	-1
f'(x)	undefined	-	0	+	undefined	-	undefined
f''(x)	undefined	+	0	+	undefined	-	undefined

a) Find the x coordinate for each local extrema (identify whether it is a min or a max). Justify your answer.

→ Rel min for f @ x = 2 because f' changes from − to + @ x = 2

→ Rel max for f @ x = 3 because f' changes from + to − @ = 3

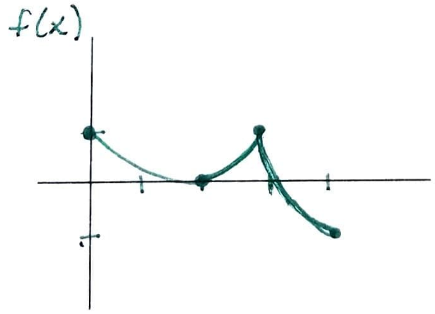
b) Find the x- coordinate for any points of inflection. Justify your answer.

f has a POI @ x = 3 because f'' changes signs @ x = 3.

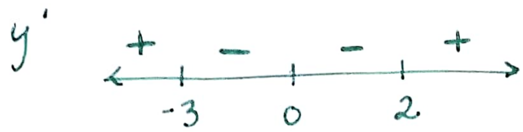
c) Where is f(x) concave down? Justify your answer.

f is cc ↓ on (3,4) because f'' is negative on this interval.

d) Sketch the graph of a function with the given characteristics. Justify your answer.



9. For $y' = x^2(x+3)(x-2)$, find the x-values of any local max and min(Identify which are maximums and which are minimums).



Rel max @ $x = -3$

f' changes from + to −

Rel min @ $x = 2$

f' changes from − to +

10. $f(x) = \frac{2}{(x-1)^2}$ Does Rolle's Theorem Apply on the interval [0,2]?

No

There is a vertical asymptote @ $x = 1$

$\therefore$ $f(x)$ is not continuous on $[0, 2]$