Notes on Curve Sketching 4.3 and 4.4

name Key

$f(x) = x^3 + 3x^2 - 9x + 18$

Step 1: Take the derivative.

$f'(x) = 3x^2 + 6x - 9$

Step 2: Find the critical values.

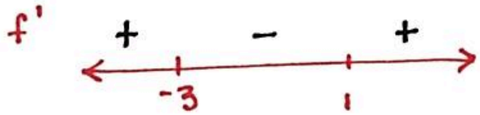
{What makes numerators and denominators =0?}

$0 = 3(x^2 + 2x - 3)$

$0 = 3(x+3)(x-1)$

CV: $x = -3, 1$

Step 3: Make a sign chart to check for increasing/decreasing.



Increasing: $(-\infty, -3) \cup (1, \infty)$

Decreasing: $(-3, 1)$

Step 4: Take the second derivative.

$f''(x) = 6x + 6$

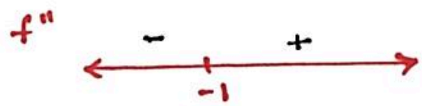
Step 5: Find the possible points of inflections.

{What makes numerators and denominators =0?}

$0 = 6(x+1)$

PPOI: $x = -1$

Step 6: Make a sign chart to check for concave up/concave down.



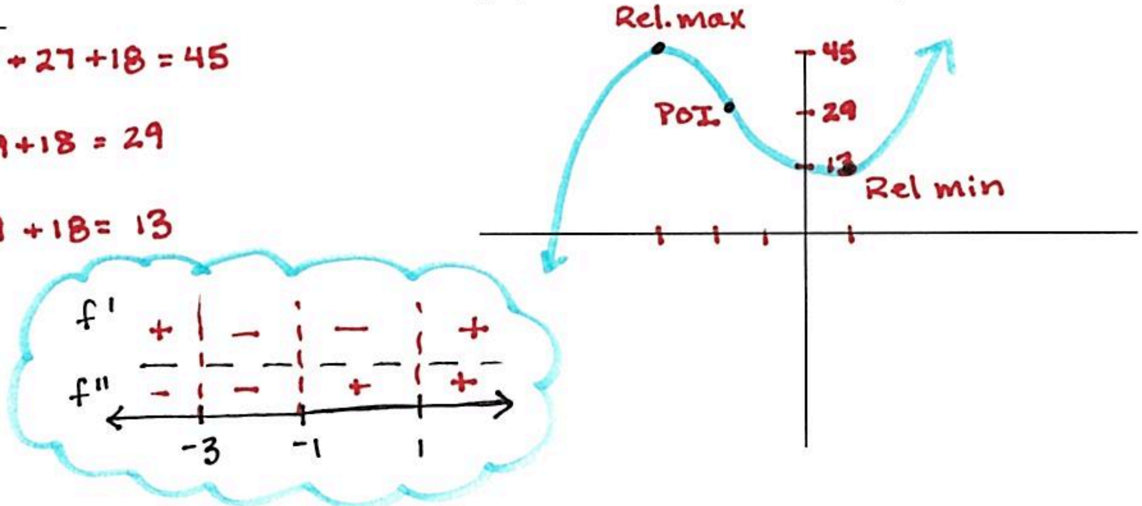
Concave Up: $(-1, \infty)$

Concave Down: $(-\infty, -1)$

Step 7: Make a table of values

x	f(x)
-3	$-27+27+27+18 = 45$
-1	$-1+3+9+18 = 29$
1	$1+3-9+18 = 13$

Step 8: make a combined sign chart and use it to sketch a graph. Label relative extrema and points of inflection.



$f(x) = x - 3x^{\frac{2}{3}}$

Step 1: Take the derivative.

$f'(x) = 1 - 2x^{-1/3} = 1 - \frac{2}{\sqrt[3]{x}}$

$= \frac{\sqrt[3]{x} - 2}{\sqrt[3]{x}}$

Step 2: Find the critical values.

{What makes numerators and denominators =0?}

$\sqrt[3]{x} - 2 = 0$ $\quad\quad$ $\sqrt[3]{x} = 0$

$\sqrt[3]{x} = 2$ $\quad\quad$ $x = 0$

$x = 8$

CV: 0, 8

Step 3: Make a sign chart to check for increasing/decreasing.

Increasing: $(-\infty, 0) \cup (8, \infty)$

Decreasing: $(0, 8)$

Step 4: Take the second derivative.

$f''(x) = \frac{2}{3}x^{-4/3}$

$= \frac{2}{3\sqrt[3]{x^4}}$

Step 5: Find the possible points of inflections.

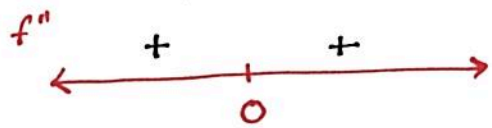
{What makes numerators and denominators =0?}

$2 \neq 0$ $\quad\quad$ $3\sqrt[3]{x^4} = 0$

$x = 0$

PPOI: $x = 0$

Step 6: Make a sign chart to check for concave up/concave down.



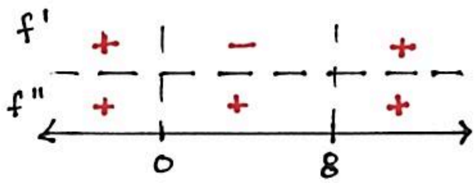
Concave Up: $(-\infty, \infty)$

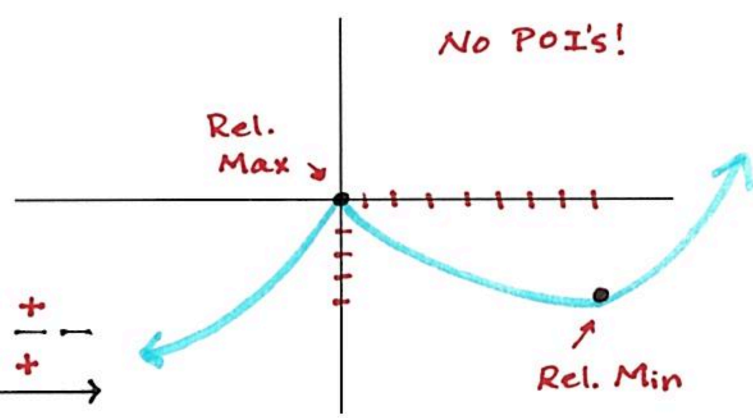
Concave Down: N/A

Step 7: Make a table of values

x	f(x)
0	0
8	$8 - 3(2)^2 = -4$

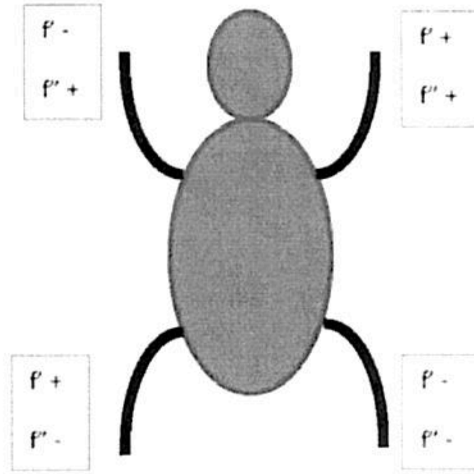
Step 8: make a combined sign chart and use it to sketch a graph. Label relative extrema and points of inflection.





Derivate Test Recap:

First Derivative:



If $f'(c) = 0$ or $f'(c)$ is undefined,

then c is a Critical Value.

If $f'(x) > 0$ on the interval (a,b), then $f(x)$ is increasing.

If $f'(x) < 0$ on the interval (a,b), then $f(x)$ is decreasing.

(First Derivative Test)

If $f'(c) = 0$ or $f'(c)$ is undefined, and $f'(x)$ changes from positive to negative at x = c, then (c, $f(c)$) is a relative maximum.

If $f'(c) = 0$ or $f'(c)$ is undefined, and $f'(x)$ changes from negative to positive at x = c, then (c, $f(c)$) is a relative minimum.

Second Derivative:

If $f''(c) = 0$ or $f''(c)$ is undefined, then c is a possible point of inflection

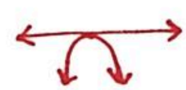
If $f''(x) > 0$ on the interval (a,b), then $f(x)$ is Concave up.

If $f''(x) < 0$ on the interval (a,b), then $f(x)$ is concave down.

If $f''(c) = 0$ or $f''(c)$ is undefined, and $f''(x)$ changes sign at x = c, then (c, $f(c)$) is a point of inflection.

(Second Derivative Test)

If $f'(c) = 0$ or $f'(c)$ is undefined, and if $f''(c) > 0$, then (c, $f(c)$) is a relative minimum.



If $f'(c) = 0$ or $f'(c)$ is undefined, and if $f''(c) < 0$, then (c, $f(c)$) is a relative maximum.