Key

p. 232 #13, #17, ~~#19~~, #21, #26, ~~#41~~, #43

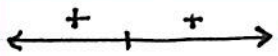
13. $f(x) = x^3 - 6x^2 + 12x$

$f' = 3x^2 - 12x + 12$

$0 = 3(x^2 - 4x + 4)$

$0 = 3(x-2)(x-2)$

CV: $x = 2$



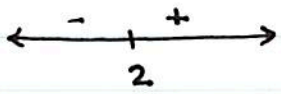
$f(x)$ is always increasing

no relative extrema

$f'' = 6x - 12$

$0 = 6(x-2)$

PPOI: $x = 2$

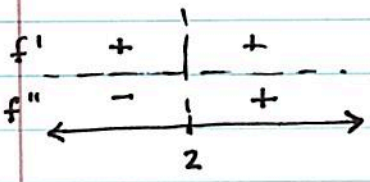


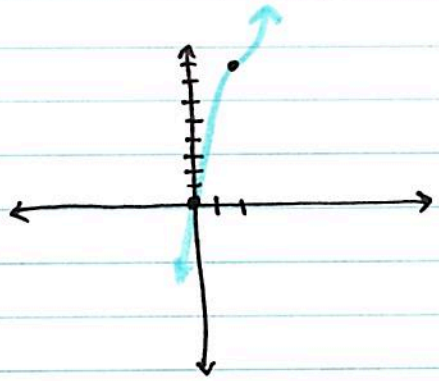
concave ↑ $(2, \infty)$

concave ↓ $(-\infty, 2)$

POI @ $x = 2$

x	f(x)
2	8





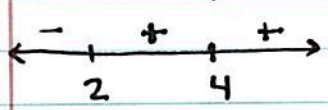
17. $f(x) = x(x-4)^3$

$f' = x \cdot 3(x-4)^2 + (x-4)^3$

$f' = (x-4)^2(3x - (x-4))$

$0 = (x-4)^2(2x-4)$

CV: $x = 4, 2$



increasing $(2, \infty)$

decreasing $(-\infty, 2)$

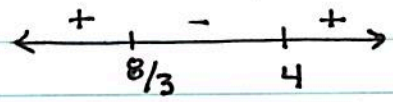
Rel min @ $x = 2$

$f'' = (x-4)^2 \cdot 2 + (2x-4) \cdot 2(x-4)$

$f'' = 2(x-4)[(x-4) + 2x - 4]$

$f'' = 2(x-4)(3x-8)$

PPOI: $x = 4, 8/3$



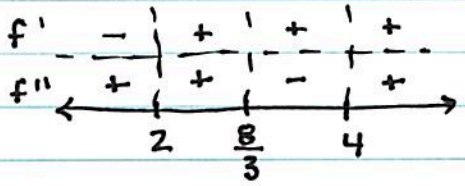
concave ↑: $(-\infty, 8/3) \cup (4, \infty)$

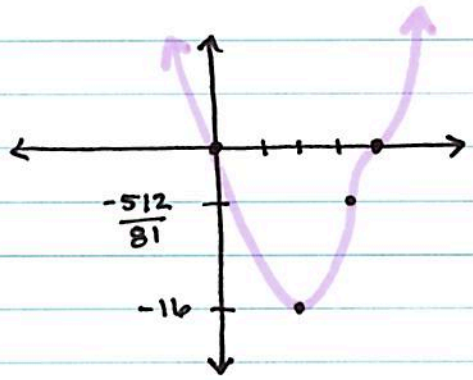
concave ↓: $(8/3, 4)$

POI @ $x = 8/3, 4$

17 continued

x	f(x)
2	-16
8/3	-512/81 (≈6.2)
4	0

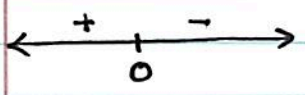




21 $f(x) = \frac{4}{x^2+1} = 4(x^2+1)^{-1}$

$f' = \frac{-4(2x)}{(x^2+1)^2}$

CV: $x = 0$



increasing $(-\infty, 0)$

decreasing $(0, \infty)$

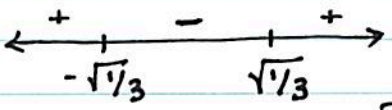
Rel. max @ $x = 0$

$f'' = \frac{(x^2+1)^2 \cdot -8 + 8x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$

$f'' = \frac{-8x^2 - 8 + 32x^2}{(x^2+1)^3}$

$= \frac{24x^2 - 8 = 0 \quad 8(3x^2-1)}{(x^2+1)^3 \neq 0}$

PPOI: $x = \pm\sqrt{1/3}$



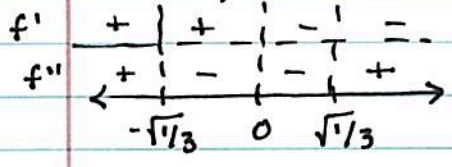
concave ↑: $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$

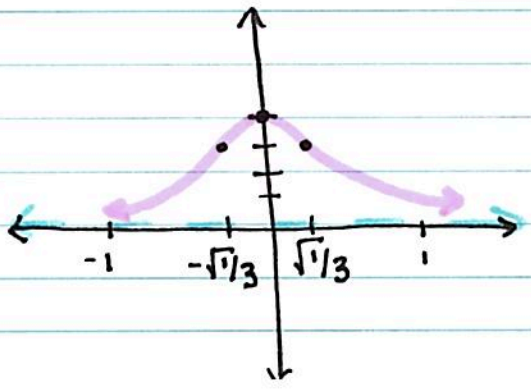
concave ↓: $(-\sqrt{1/3}, \sqrt{1/3})$

POI @ $x = \pm\sqrt{1/3}$

x	f(x)
$-\sqrt{1/3}$	3
0	4
$\sqrt{1/3}$	3

HA @ $y = 0$





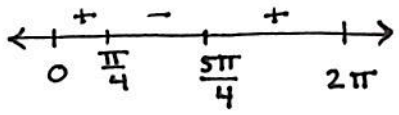
26. $f(x) = \sin x + \cos x$

$f' = \cos x - \sin x$

$0 = \cos x - \sin x$

$\sin x = \cos x$

CV: $x = \frac{\pi}{4}, \frac{5\pi}{4}$



increasing: $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$

decreasing: $(\frac{\pi}{4}, \frac{5\pi}{4})$

Rel max @ $x = \frac{\pi}{4}$

Rel min @ $x = \frac{5\pi}{4}$

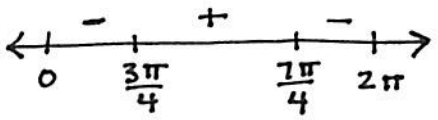
x	$f(x)$
0	1
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{3\pi}{4}$	0
$\frac{5\pi}{4}$	$-\sqrt{2}$
$\frac{7\pi}{4}$	0
2π	1

$f'' = -\sin x - \cos x$

$0 = -\sin x - \cos x$

$\sin x = -\cos x$

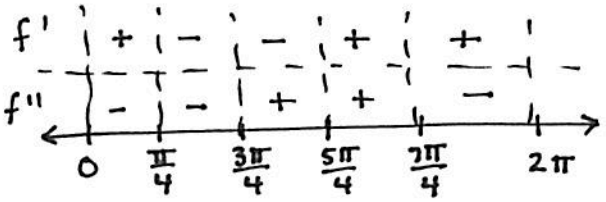
PPOI: $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

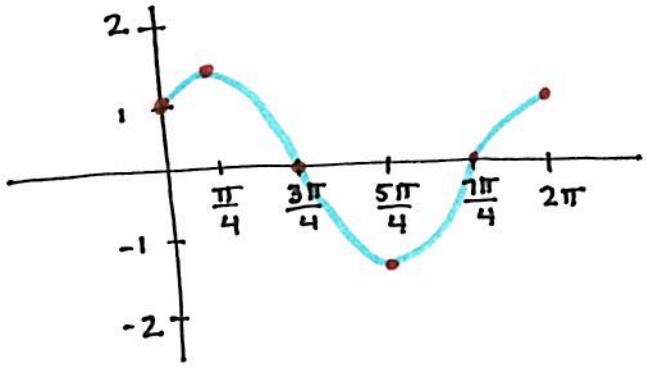


Concave ↑: $(\frac{3\pi}{4}, \frac{7\pi}{4})$

Concave ↓: $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

POI @ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$



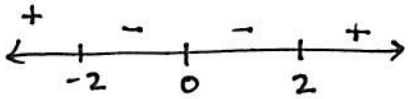


43) $f(x) = x + \frac{4}{x}$

$f' = 1 - \frac{4}{x^2}$

$= \frac{x^2 - 4 = 0}{x^2 = 0}$

CV: $x = \pm 2, \emptyset$ 0 is a vertical asymptote



increasing $(-\infty, -2) \cup (2, \infty)$

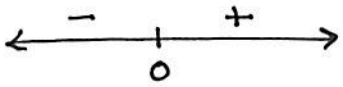
decreasing $(-2, 0) \cup (0, \infty)$

Rel max @ $x = -2$

Rel min @ $x = 2$

$f'' = \frac{8}{x^3}$

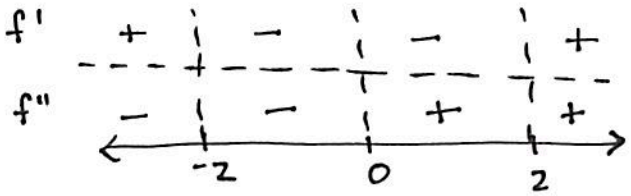
PPoI @ $x = \emptyset$ It is a V. Asymptote



Concave ↑: $(0, \infty)$

Concave ↓: $(-\infty, 0)$

no POI



$f(x) = \frac{x^2 + 4}{x}$

slant asymptote!

$$x \overline{)\, x^2 + 0x + 4} \quad (x)$$

$x^2 + 0x$

$0 + 4$

SA @ $y = x$

x	f(x)
-2	-4
0	und
2	4