CALCULUS AB WORKSHEET ON AREA AND VOLUME

Work the following on notebook paper. Do **not** use your calculator.

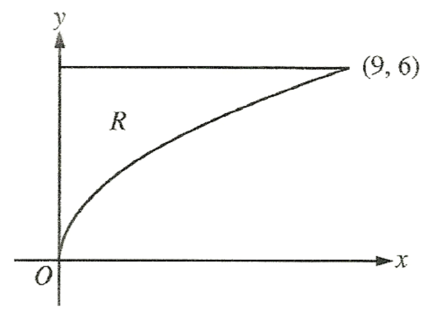
1. Let R be the region bounded by the graphs of $y = 4 - x^2$ and $y = x + 2$.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Write, but do not evaluate, an integral expression for the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

2. Let R be the region in the first quadrant bounded by the graphs of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure on the right.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

3. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.

(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

4. Let R be the region in the first quadrant bounded by the graphs of $y = x$, $y = \dfrac{1}{x^2}$, the x-axis and the vertical line $x = 3$.

(a) Find the area of the region R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are rectangles with height five times the length of the base. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

Answers to Worksheet on Area and Volume

1. $4-x^2=x+2$ at $x=-2$ and $x=1$

(a) $A=\int_{-2}^{1}\left(\left(4-x^2\right)-\left(x+2\right)\right)dx=\ldots=\frac{9}{2}$

(b) $V=\int_{-2}^{1}\left(2-x^2-x\right)^2dx$

(c) $V=\pi\int_{-2}^{1}\left(\left(4-x\right)^2-\left(2+x^2\right)^2\right)dx$

2. (a) $A=\int_{0}^{9}\left(6-2\sqrt{x}\right)dx=\ldots=18$

(b) $V=\pi\int_{0}^{9}\left(\left(7-2\sqrt{x}\right)^2-1^2\right)dx$

(c) $V=\int_{0}^{6}\frac{3y^4}{16}dy$

3. (a) $A=\int_{0}^{4}\sqrt{x}dx=\ldots=\frac{16}{3}$

(b) $\int_{0}^{h}\sqrt{x}dx=\frac{1}{2}\left(\frac{16}{3}\right)$

$h=4^{2/3}$

(c) $V=\pi\int_{0}^{4}\left(\sqrt{x}\right)^2dx=\ldots=8\pi$

(d) $\pi\int_{0}^{k}\left(\sqrt{x}\right)^2dx=\frac{1}{2}\left(8\pi\right)$

$k=\sqrt{8}$

4. $x=\frac{1}{x^2}$ at $x=1$

(a) $A=\int_{0}^{1}x\,dx+\int_{1}^{3}\frac{1}{x^2}dx=\ldots=\frac{7}{6}$

(b) $V=\int_{0}^{1}5x^2\,dx+\int_{1}^{3}\frac{5}{x^4}dx=\ldots=\frac{265}{81}$

(c) $V=\pi\int_{0}^{1}\left(2^2-\left(2-x\right)^2\right)dx+\pi\int_{1}^{3}\left(2^2-\left(2-\frac{1}{x^2}\right)^2\right)dx$