AB Calculus (Calculator OK)
Average vs. Instantaneous Rate of Change name Key

1. Suppose $g(t) = 1 - t^2$.
a. Find the slope of the line secant to g(t) from t = -1 to t = 3.

$$\frac{g(3) - g(-1)}{3-(-1)} = \frac{-8-0}{4} = -2$$

b. Find the average rate of change of g on the interval [-1, -.5]

$$\frac{g(-.5) - g(-1)}{-.5-(-1)} = \frac{3}{2}$$

c. Suppose g represents the position (in cm) of an object at time t seconds. Find the average velocity of the object on the time interval starting at t = -1 and lasting .1 seconds. Include units with your answer.

$$\frac{g(-.9) - g(-1)}{-.9-(-1)} = 1.9 \text{ cm/sec}$$

d. If g(t) gives the temperature (°C) of a liquid at time t (sec), find the average rate of change of the temperature between t = 18 seconds and t = 21 seconds. Include units with your answer.

$$\frac{g(21) - g(18)}{21-18} = -39 \text{ °C/sec}$$

f. Find the instantaneous rate of change of g at t = -1.

$$\lim_{t \to -1} \frac{1-t^2-0}{t-(-1)} = \lim_{t \to -1} \frac{(1-t)\cancel{(1+t)}}{\cancel{t+1}}$$

$$g'(-1) = 2$$

g. Find the slope of the line tangent to g(t) at t = 0. Then find the slope of the line normal(perpendicular) to the tangent line at t = 0.

$$\lim_{t \to 0} \frac{1-t^2-1}{t-0} = \lim_{t \to 0} \frac{-t^2}{t} = 0$$

tangent line - 0

normal line - und

3. a)Use the limit definition to find the derivative of $f(x) = x^2 - 2x$

$$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - (x^2-2x)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}}{h}$$

$$\lim_{h \to 0} \frac{h(2x+h-2)}{h} = 2x-2$$

b) Use the alternate form of the definition to find f'(2).

$$\lim_{x \to 2} \frac{x^2-2x-0}{x-2} = \lim_{x \to 2} \frac{x(x-2)}{x-2} = 2$$

4. a) Use the limit definition to find the slope of the tangent line of $g(x) = \frac{1}{x+2}$.

$$\lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} = \lim_{h \to 0} \frac{x+2-(x+h+2)}{(x+h+2)(x+2)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-\cancel{h}}{(x+2)(x+h+2)\cancel{h}}$$

$$= \frac{-1}{(x+2)^2}$$

b) Use the alternate form of the definition to find g'(2).

$$\lim_{x \to 2} \frac{\frac{1}{x+2} - \frac{1}{4}}{x-2} = \lim_{x \to 2} \frac{4-(x+2)}{4(x+2)} \cdot \frac{1}{x-2}$$

$$= \lim_{x \to 2} \frac{\overset{-1}{\cancel{2-x}}}{4(x+2)\cancel{(x-2)}} = \frac{-1}{16}$$

5. The graph of the function k(x) is drawn below. Use it to answer the following questions.

a. On what interval(s) is k(x) positive? $(-6,-1) \cup (4.5, \infty)$

b. On what interval(s) is k(x) < 0? $(-\infty,-6) \cup (1, 4.5)$

c. On what interval(s) is k(x) increasing? $(-\infty,-3) \cup (1,\infty)$

d. On what interval(s) is k(x) decreasing? $(-3,1)$

e. On what interval(s) does the line tangent to k(x) have a positive slope? $(-\infty,-3) \cup (1,\infty)$

g. On what interval(s) does the line tangent to k(x) have a negative slope? $(-3,1)$

h. For which value(s) of x does the line tangent to k(x) have a slope of zero. @ $x = -3, 1$