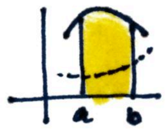
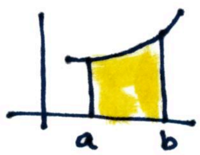
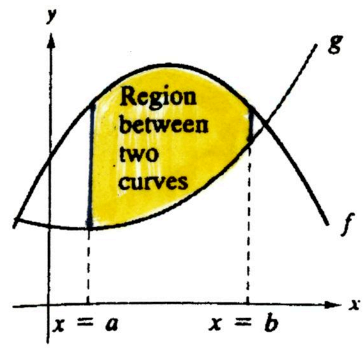
Calculus
Notes – 6.1 Area of a Region Between Two Curves

1. Use the diagram below to write an expression that could be used to find the area between the graphs of f and g on the interval $[a,b]$:

$\int_a^b f(x)\,dx =$ 

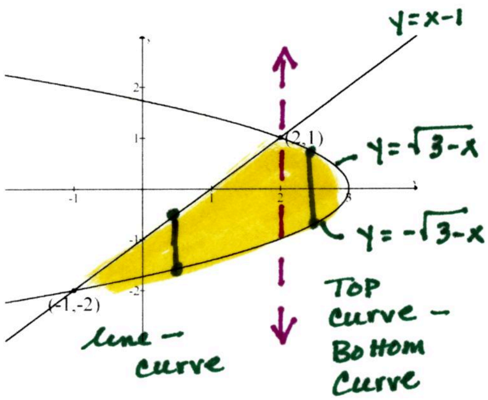
$\int_a^b g(x)\,dx =$ 



$\int_a^b [f(x) - g(x)]\,dx$

= AREA BETWEEN 2 CURVES

2. Find the area of the region bounded by the graphs of $y^2 = 3 - x$ and $y = x - 1$ $\quad y = \pm\sqrt{3-x}$



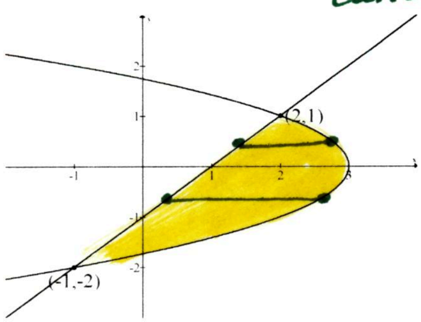
Vertically:

$$\int_{-1}^{2} [(x-1) - (-\sqrt{3-x})]\,dx$$

$$+$$

$$\int_{2}^{3} [(\sqrt{3-x}) - (-\sqrt{3-x})]\,dx$$

$$= 4.5$$



always (curve - line)

Horizontally: line: $x = y+1$

curve: $x = 3-y^2$

$$\int_{-2}^{1} [(3-y^2) - (y+1)]\,dy = 4.5$$

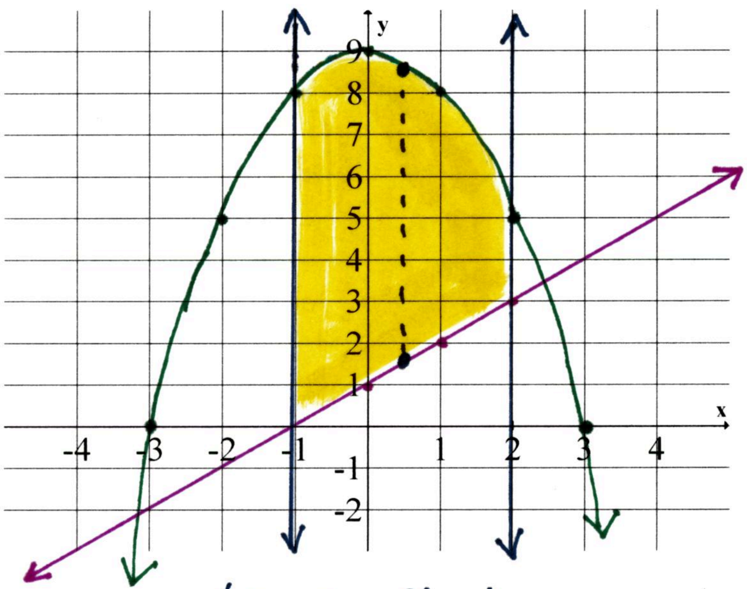
$A = \int_{x_1}^{x_2} [(\text{Top Curve}) - (\text{Bottom Curve})]\,dx$ for vertical rectangles

$A = \int_{y_1}^{y_2} [(\text{Right Curve}) - (\text{Left Curve})]\,dy$ for horizontal rectangles

ALWAYS!
(It doesn't matter where the functions are.)

Example Problems:

* 1. $y = x + 1$ $y = 9 - x^2$ $x = -1$ $x = 2$



(curve - line)

$$\int_{-1}^{2} [(9-x^2) - (x+1)]\, dx$$

$$9x - \frac{x^3}{3} - \frac{x^2}{2} - x \Big|_{-1}^{2}$$

$$(18 - \frac{8}{3} - 2 - 2) - (-9 + \frac{1}{3} - \frac{1}{2} + 1)$$

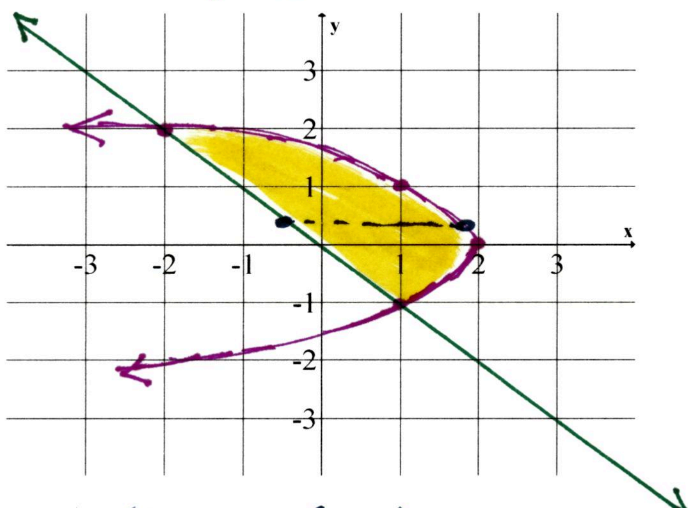
$$(14 - \frac{8}{3}) - (-8 + \frac{1}{3} - \frac{1}{2})$$

$$22 - 3 + \frac{1}{2}$$

$$\frac{132 - 14 + 3}{6} = \frac{121}{6}$$

2. $x + y^2 = 2$ $y + x = 0$

$y = \pm\sqrt{2-x}$ $y = -x$



(curve - line)

$x = 2 - y^2$ $x = -y$

$$\int_{-1}^{2} [(2-y^2) - (-y)]\, dy$$

$$2y - \frac{y^3}{3} + \frac{y^2}{2} \Big|_{-1}^{2}$$

$$(4 - \frac{8}{3} + 2) - (-2 + \frac{1}{3} + \frac{1}{2})$$

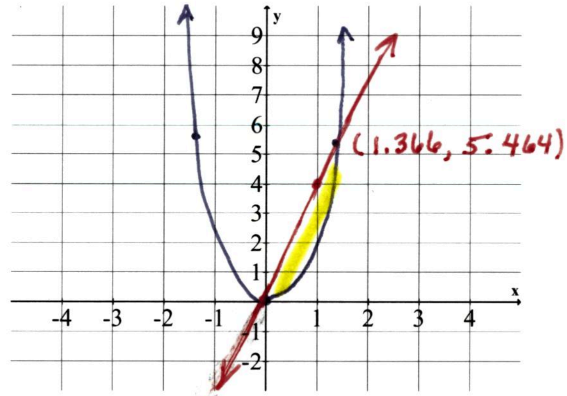
$$(6 - \frac{8}{3}) - (-2 + \frac{1}{3} + \frac{1}{2})$$

$$8 - 3 - \frac{1}{2} = \frac{9}{2}$$

3. (Use a calculator for this one.)

Let R be the region in the first quadrant enclosed by the graphs of $y = e^{(x^2)} - 1$ and $y = 4x$.

(a) Find the area of R.



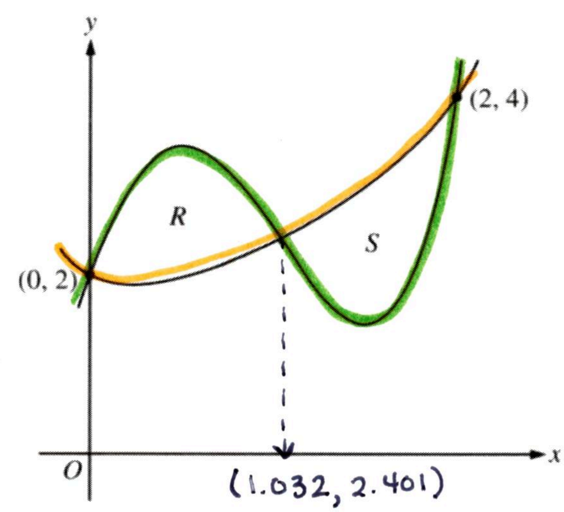
$$\int_0^{1.366} [(4x) - (e^{x^2} - 1)]\, dx$$

$= 2.087$

Steps for finding area:
1. Graph the functions.
2. Find the intersection points. (If using a calculator, store the bounds in your calculator.) [x → STO → ALPHA → A]
3. Decide if you want to find the area using dx or dy.
4. Set up the integral(s) using the intersection points as bounds. Remember Top-Bottom for dx, or Right – Left for dy.
5. a. If integrating by hand, take the antiderivative and use FTC 1 to find the area.
b. If using a calculator, use Math 9 to find the area.
6. Check to see if your answer makes sense.

4.

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.



Find the area of region R.

$$R = \int_0^{1.032} (g - f)\, dx = 0.997$$

Find the area of region S.

$$S = \int_{1.032}^{2} (f - g)\, dx = 1.006 \text{ or } 1.007$$

HWK: Area Between Two Curves

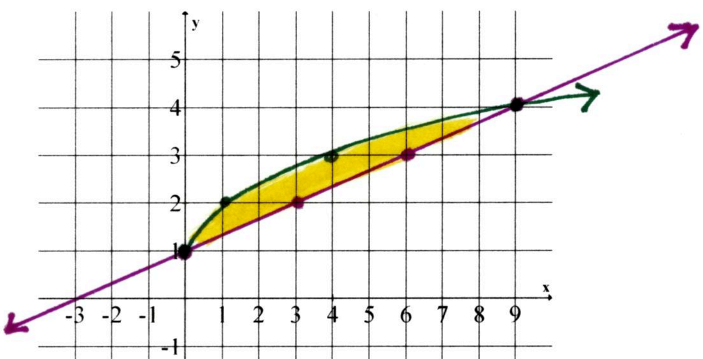
AB Calculus

name Key

For each of the following problems:

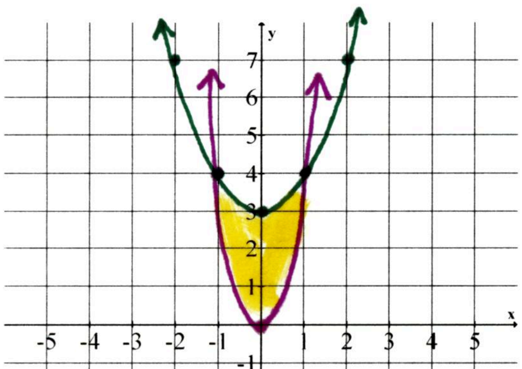
a) Sketch the region enclosed by the given curves(without a calculator).
b) Decide whether to integrate with respect to x or y.
c) Determine your limits of integration.
d) Find the area of the region(You may use a calculator on starred problems).

*1. $y=1+\sqrt{x}$ $\quad y=\dfrac{3+x}{3}$ $=1+\frac{1}{3}x$



$\int_0^9 \left[(1+\sqrt{x})-\left(\frac{3+x}{3}\right)\right]dx = 4.5$

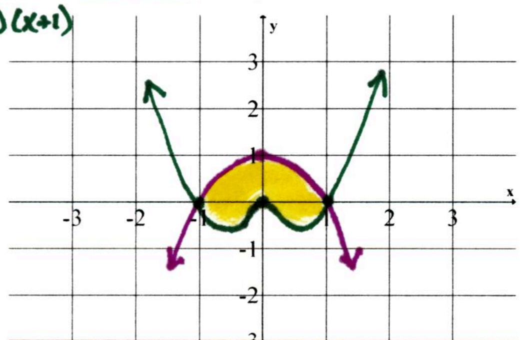
*2. $y=x^2+3$ $\quad y=4x^2$



$\int_{-1}^{1}[x^2+3-4x^2]dx = 4$

*3. $y=x^4-x^2$ $\quad y=1-x^2$

$y=x^2(x-1)(x+1)$

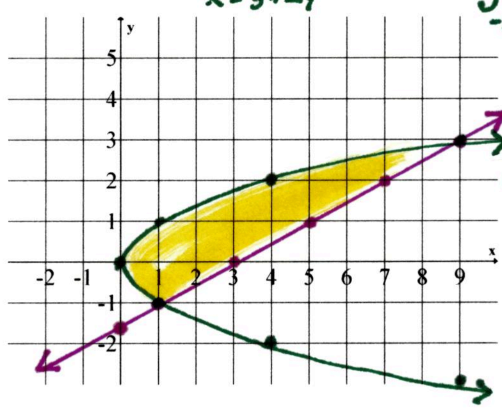


$\int_{-1}^{1}\left[(1-x^2)-(x^4-x^2)\right]dx = 1.6$

$y=\pm\sqrt{x}$ $\quad y=\frac{1}{2}x-\frac{3}{2}$

4. $y^2=x$ $\quad x-2y=3$

$x=3+2y$



$\int_{-1}^{3}[3+2y-y^2]dy$

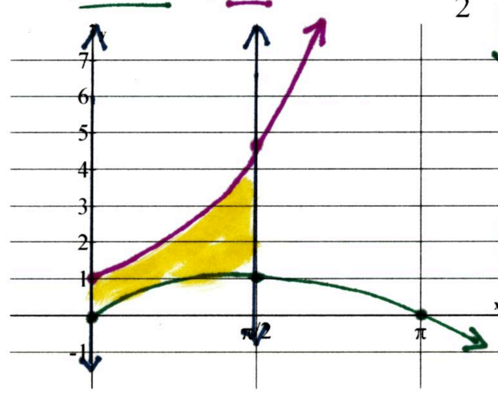
$3y+y^2-\frac{y^3}{3}\Big|_{-1}^{3}$

$(9+9-9)-(-3+1+$

$9+3-1-\frac{1}{3}$

$10\frac{2}{3}$

5. $y=\sin x$ $\quad y=e^x$ $\quad x=0$ $\quad x=\dfrac{\pi}{2}$



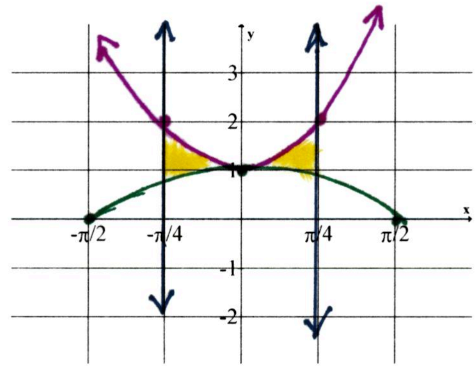
$\int_0^{\frac{\pi}{2}}(e^x-\sin x)dx$

$e^x+\cos x\Big|_0^{\frac{\pi}{2}}$

$(e^{\frac{\pi}{2}}+0)-(1+1)$

$e^{\frac{\pi}{2}}-2$

6. $y=\cos x$ $\quad y=\sec^2 x$ $\quad x=\dfrac{-\pi}{4}$ $\quad x=\dfrac{\pi}{4}$



$2\int_0^{\frac{\pi}{4}}(\sec^2 x-\cos x)$

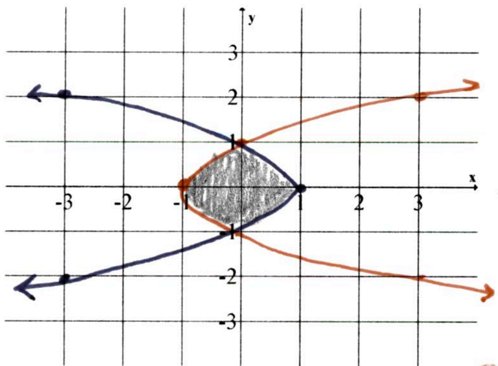
$2\tan x-2\sin x\Big|$

$2(1)-2\left(\frac{\sqrt{2}}{2}\right)-($

$2-\sqrt{2}$

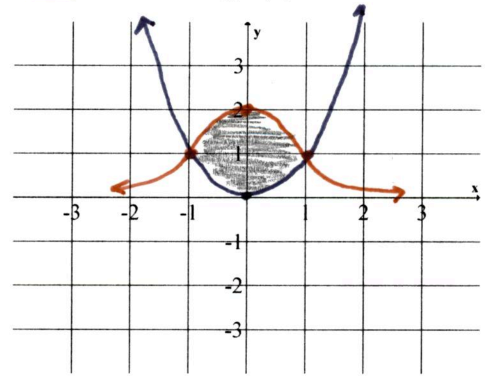
$y = \pm\sqrt{1-x}$ $y = \pm\sqrt{x+1}$

*7. $x = 1 - y^2 \quad x = y^2 - 1$



$\int_{-1}^{1} [(1-y^2) - (y^2-1)]\,dy = \frac{8}{3}$

8. $y = x^2 \qquad y = \dfrac{2}{x^2+1}$



$\int_{-1}^{1} \left[\frac{2}{x^2+1} - x^2\right] dx$

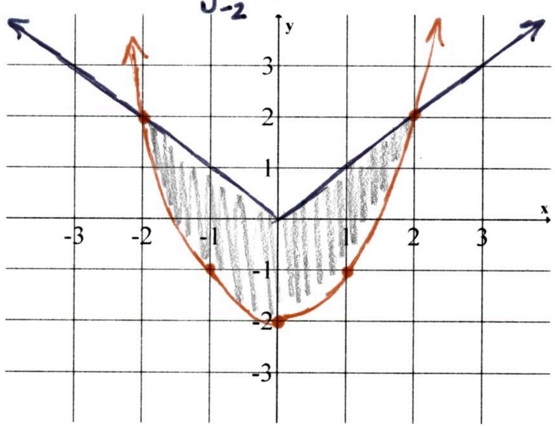
$\left(2\tan^{-1}(x) - \frac{x^3}{3}\right)\Big|_{-1}^{1}$

$\left(2\left(\frac{\pi}{4}\right) - \frac{1}{3}\right) - \left(2\left(\frac{-\pi}{4}\right) + \frac{1}{3}\right)$

$= \pi - \frac{2}{3}$

*9. $y = |x| \qquad y = x^2 - 2$

$\int_{-2}^{2} [|x| - (x^2-2)]\,dx = \frac{20}{3}$



*10 Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

(a) Find the area of R.   ↳ y_1 ↳ y_2

$\int_{0}^{1.940} \left[4x - x^3 + 1 - \frac{3}{4}x\right] dx$

$= 4.5146$